Advanced Computer Graphics
Mesh Processing

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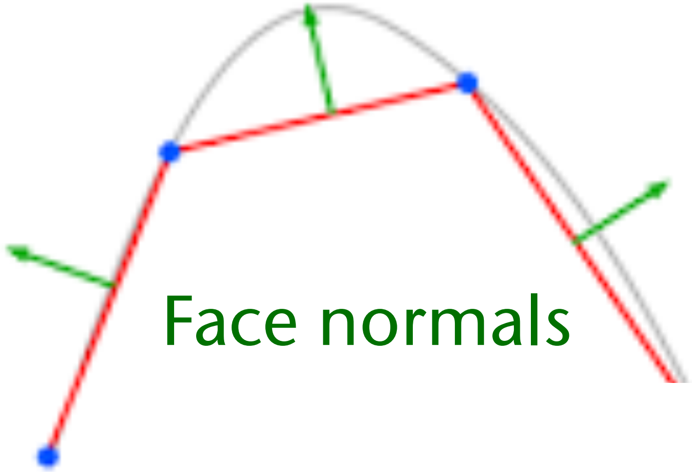
Vertex Normals

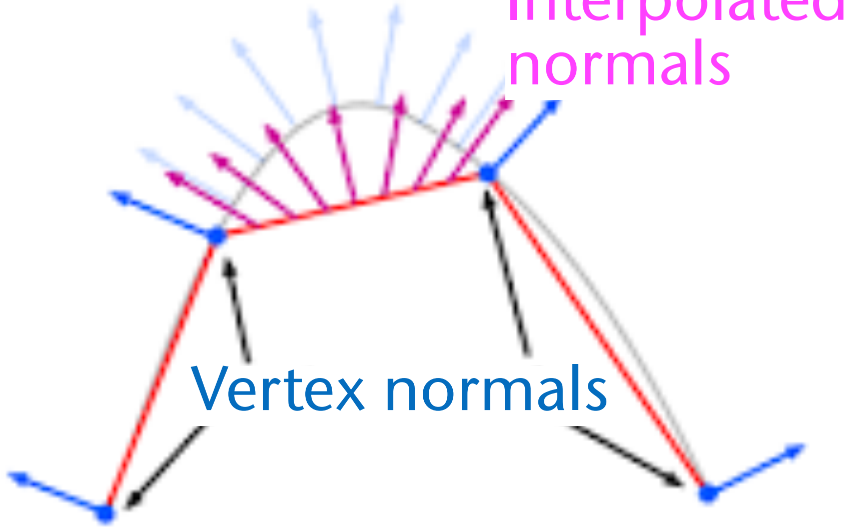
- Polygonal surfaces are (usually) just a linear approximation of smooth surfaces
- Wanted: good vertex normals
  - "Good" = as close as possible to true normals
  - Ansatz: compute vertex normal $\mathbf{n}_0$ at vertex V_0 as

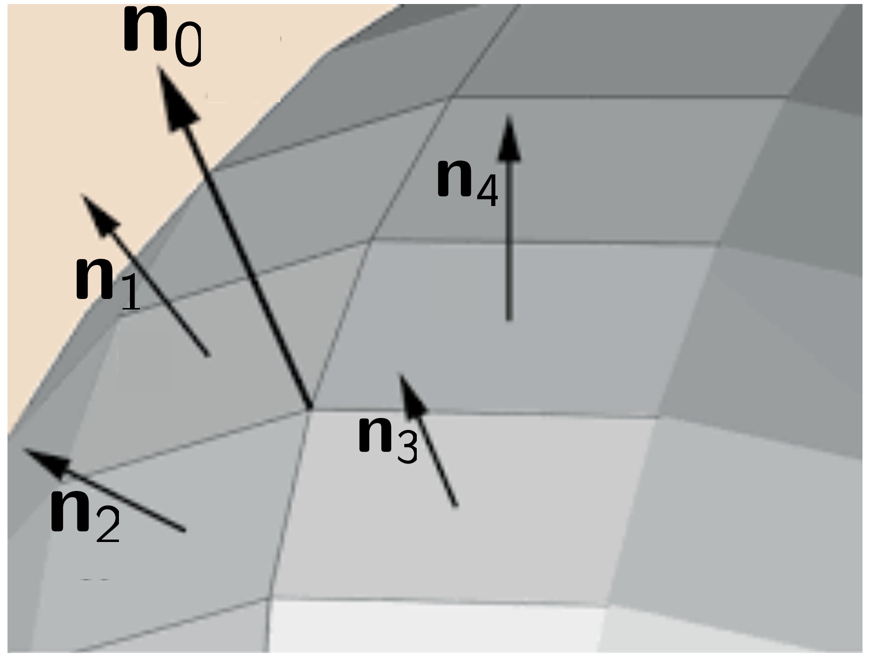
$$\mathbf{n}_0 = \sum_{i=1}^{k} w_i \mathbf{n}_i$$

    where $\mathbf{n}_i$ = normal of face given by $V_0 V_i V_{i+1}$,
    w_i = some weight

  - Question: which weights give best normals?







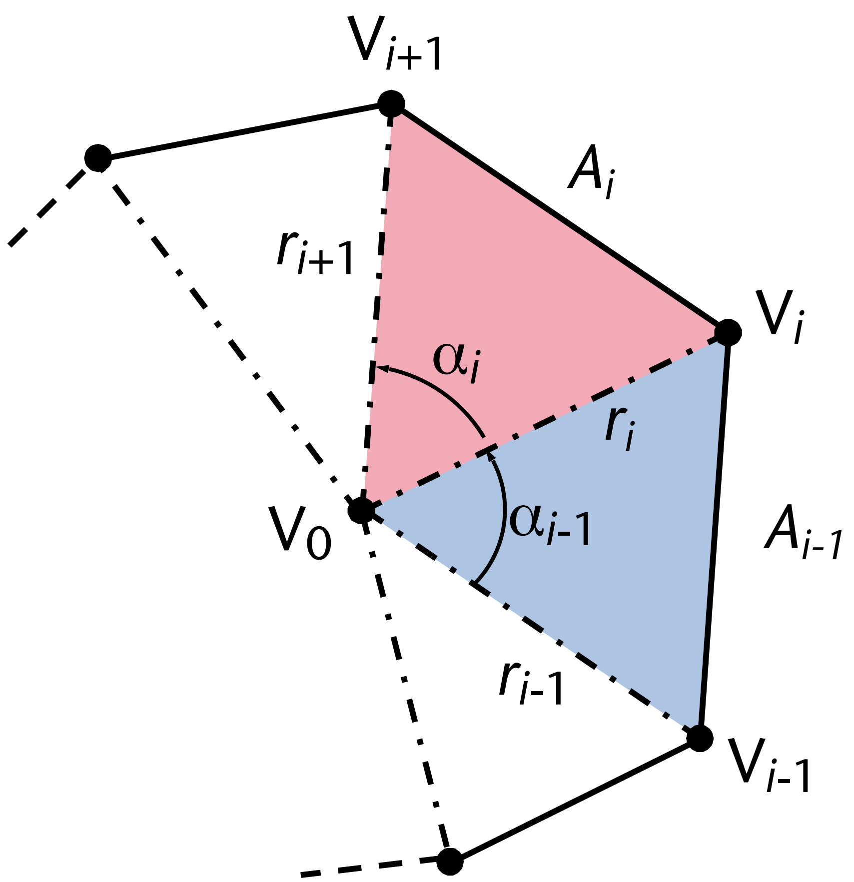
Weights That Have Been Proposed in the Literature

- No weights, i.e. $w_i = 1$
- $w_i = A_i$ (area), $w_i = \alpha_i$,

  $w_i = \frac{1}{r_i r_{i+1}}$ with $r_i := \|V_i - V_0\|$
- Best (so far) [Nelson Max]:

$$w_i = \frac{\sin(\alpha_i)}{r_i r_{i+1}}$$

  - Gives *provably* correct normals for polyhedra inscribed in sphere (= degree 2 surface)
  - Smallest RMSE almost everywhere for polygonal approximations of polynomial surface of degree 3

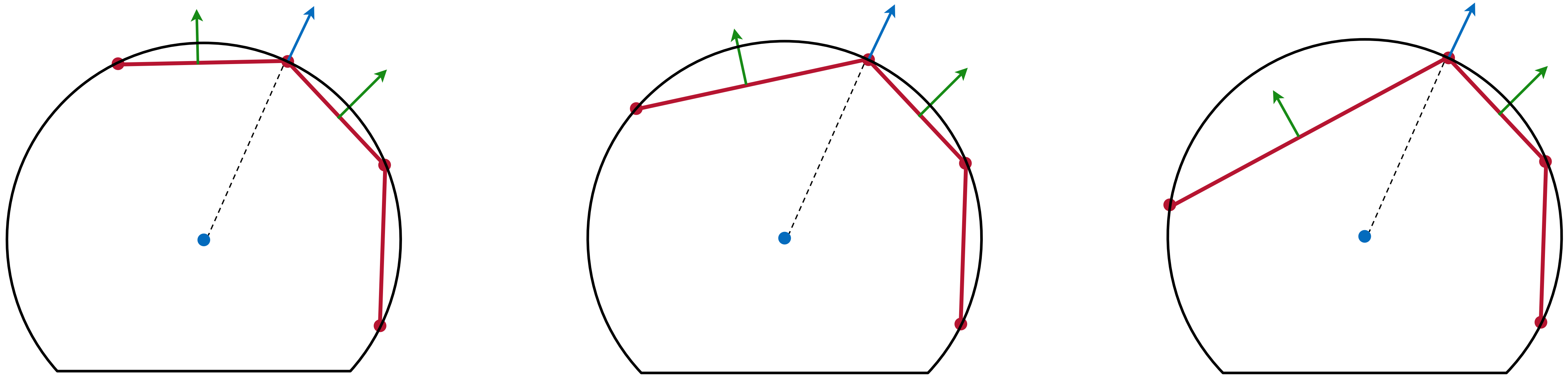


Weights	RMSE
One (no weights)	7.3 – 3.7
A_i	6.5 – 2.8
α_i	10.7 – 3.4
$\frac{1}{r_i r_{i+1}}$	7.3 – 5.1
Best $\left(\frac{\sin(\alpha_i)}{r_i r_{i+1}} \right)$	3.0 – 1.5

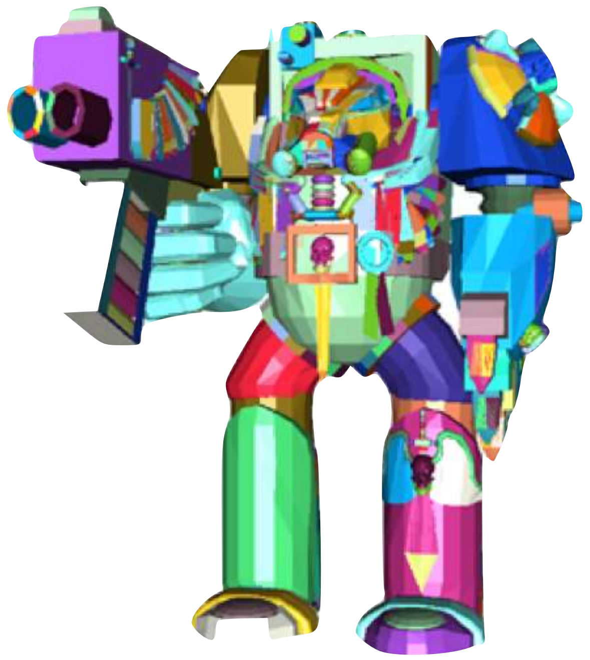
- Practical computation:
  - Remember: $(V_i - V_0) \times (V_{i+1} - V_0) = \sin(\alpha_i) r_i r_{i+1} \mathbf{n}_i$
  - In practice, this allows for easier computation of the vertex normal:

$$\mathbf{n}_0 = \sum_{i=1}^{k} \frac{(V_i - V_0) \times (V_{i+1} - V_0)}{(V_i - V_0)^2 \, (V_{i+1} - V_0)^2}$$

- Geometric intuition why *longer* faces should have *smaller* weights:

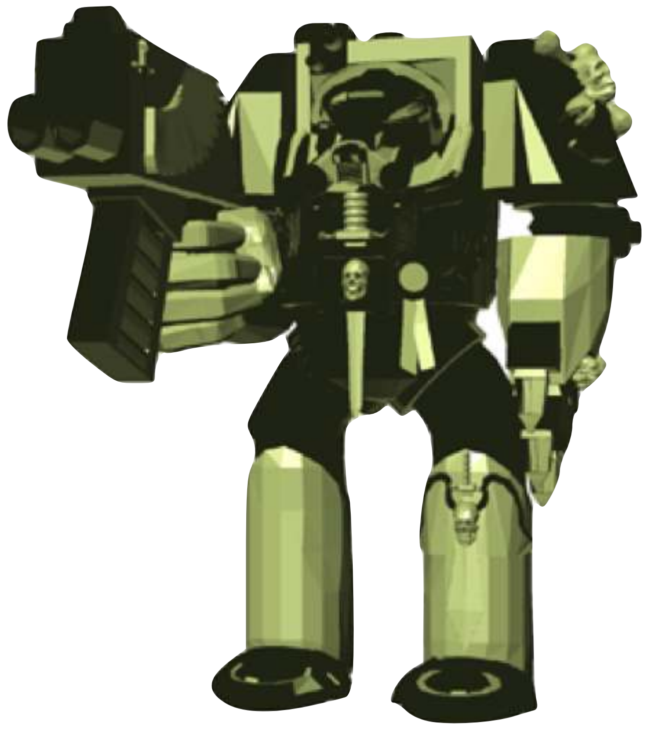


Consistent Normal Orientation for Meshes



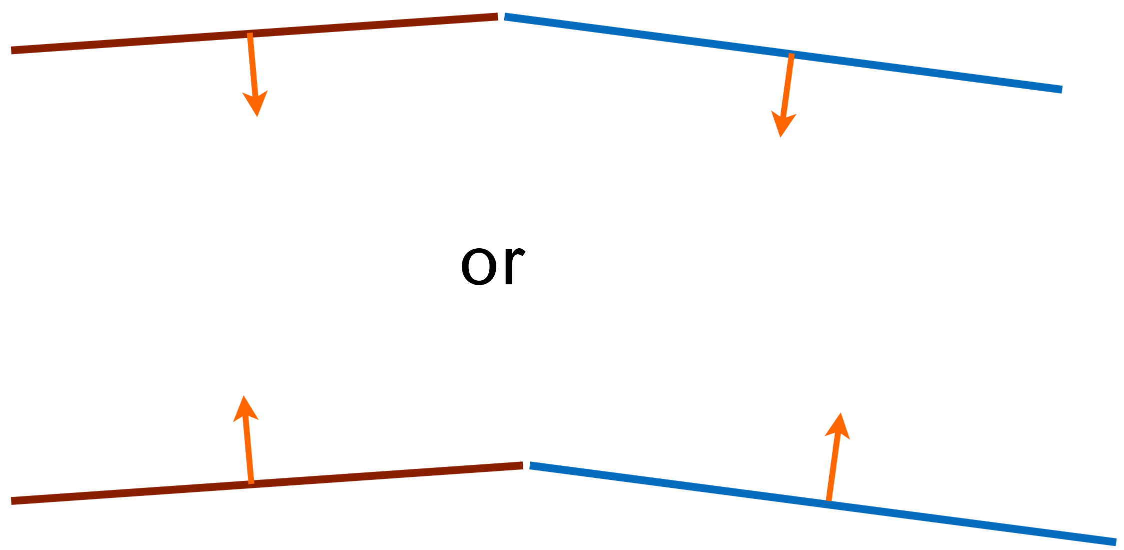
- Problem:
  - Many models consist of many unconnected patches (in particular those created with modelling tools)
  - Patches do not necessarily have consistent orientation
- Bad consequences:
  - Two-sided lighting is necessary (slightly slower than one-sided lighting)
  - BSP representation of polyhedra is difficult to construct with inconsistent normals
  - And many more …

double-sided lighting



single-sided lighting

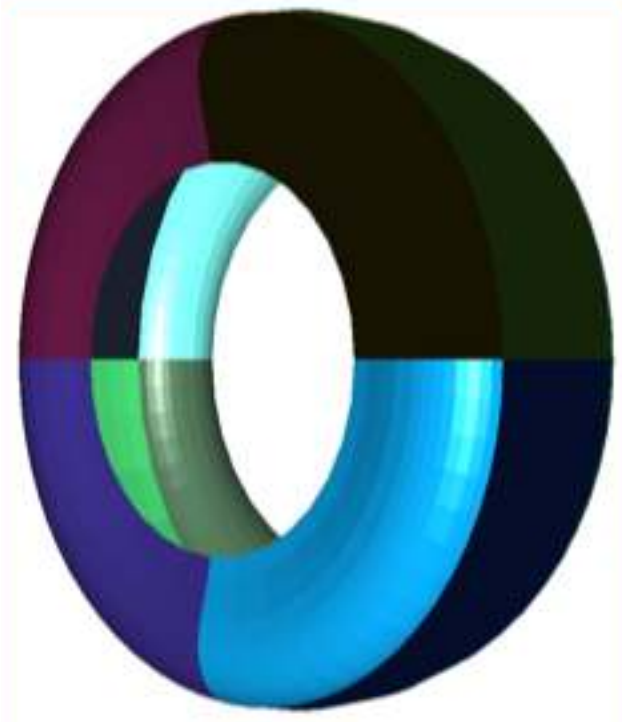
- Idea for a solution: *boundary coherence* = patches with common boundaries should be oriented consistently

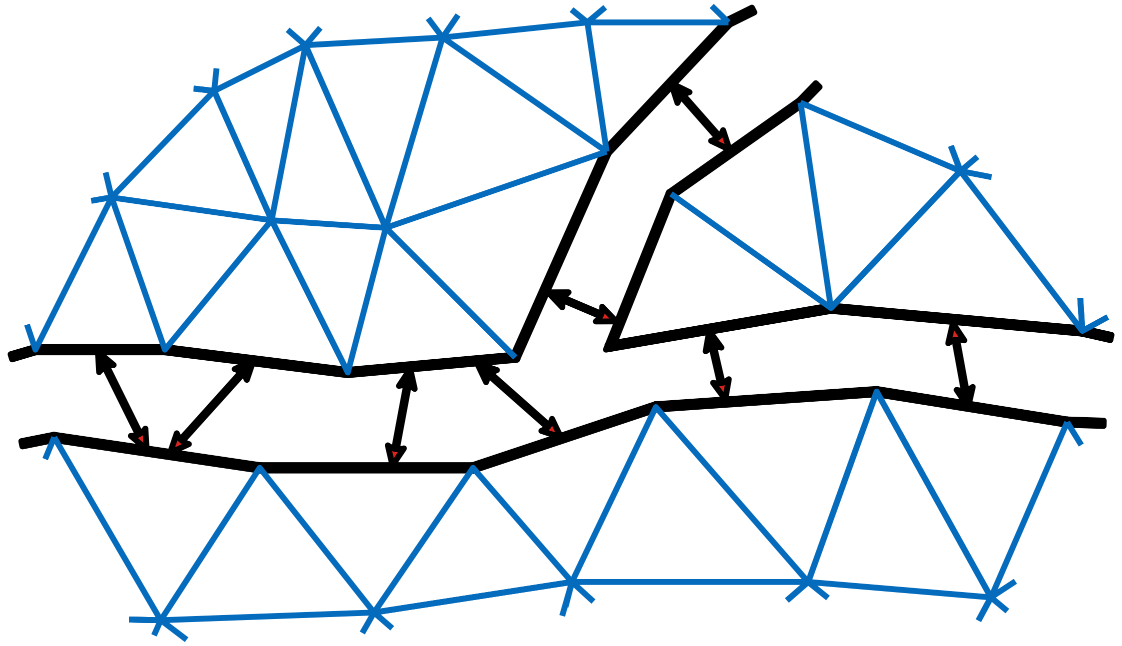


- This is fairly straight-forward to implement, provided we have *complete neighborhood information* (topology)
  - And assuming the mesh is closed

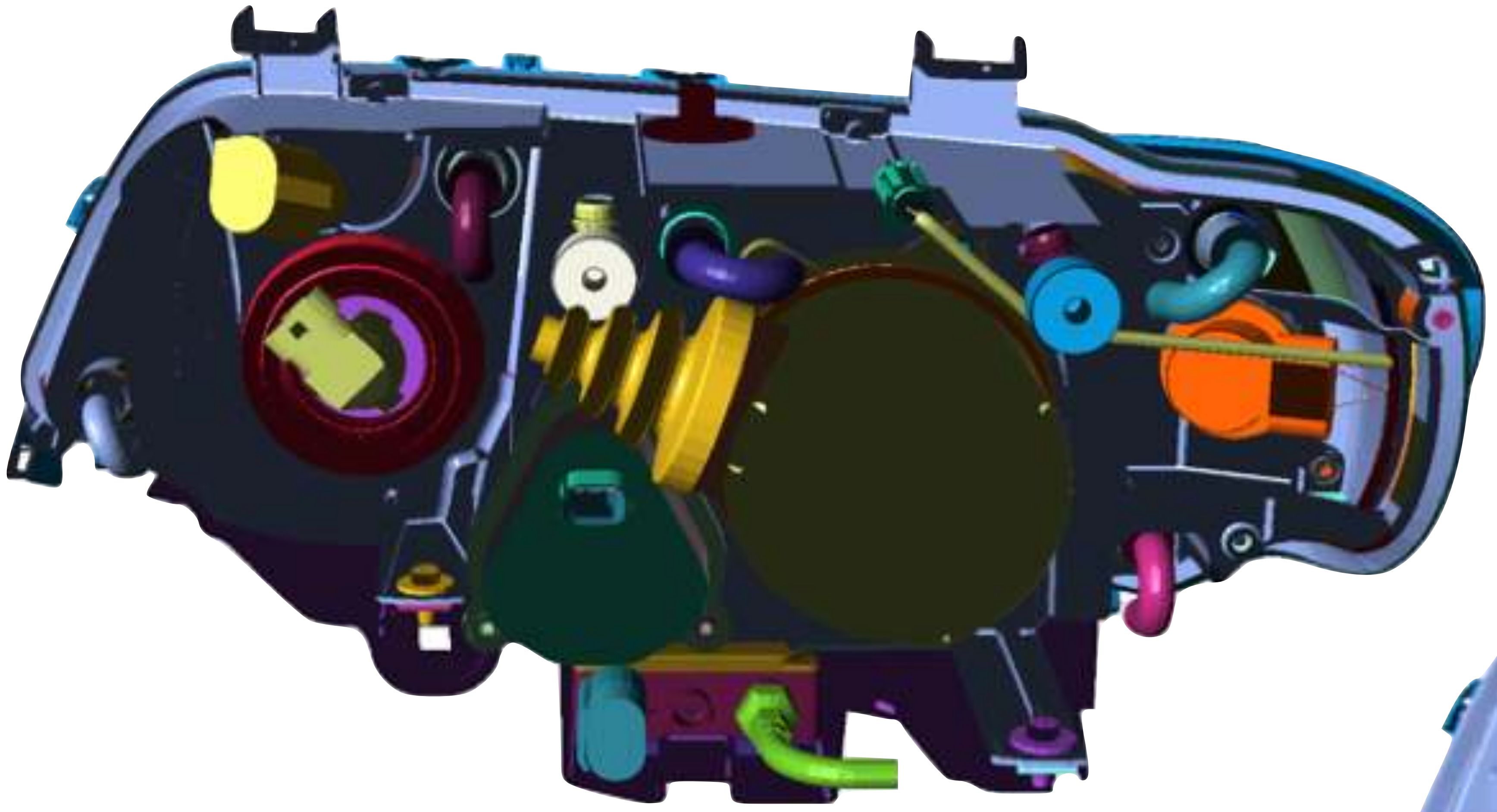
General Procedure

1. Detect edges incident to only 1 polygon (boundary edges), or incident to more than 2 polygons (non-manifold edges)
2. Partition mesh into 2-manifold patches
3. Orient normals consistently *within* each patch (propagate consistent normal direction from one polygon to the next throughout a patch using BFS)
4. Determine patch-patch boundaries close to each other (which are "meant" to be connected)
5. Propagate normal orientations across those boundaries, too



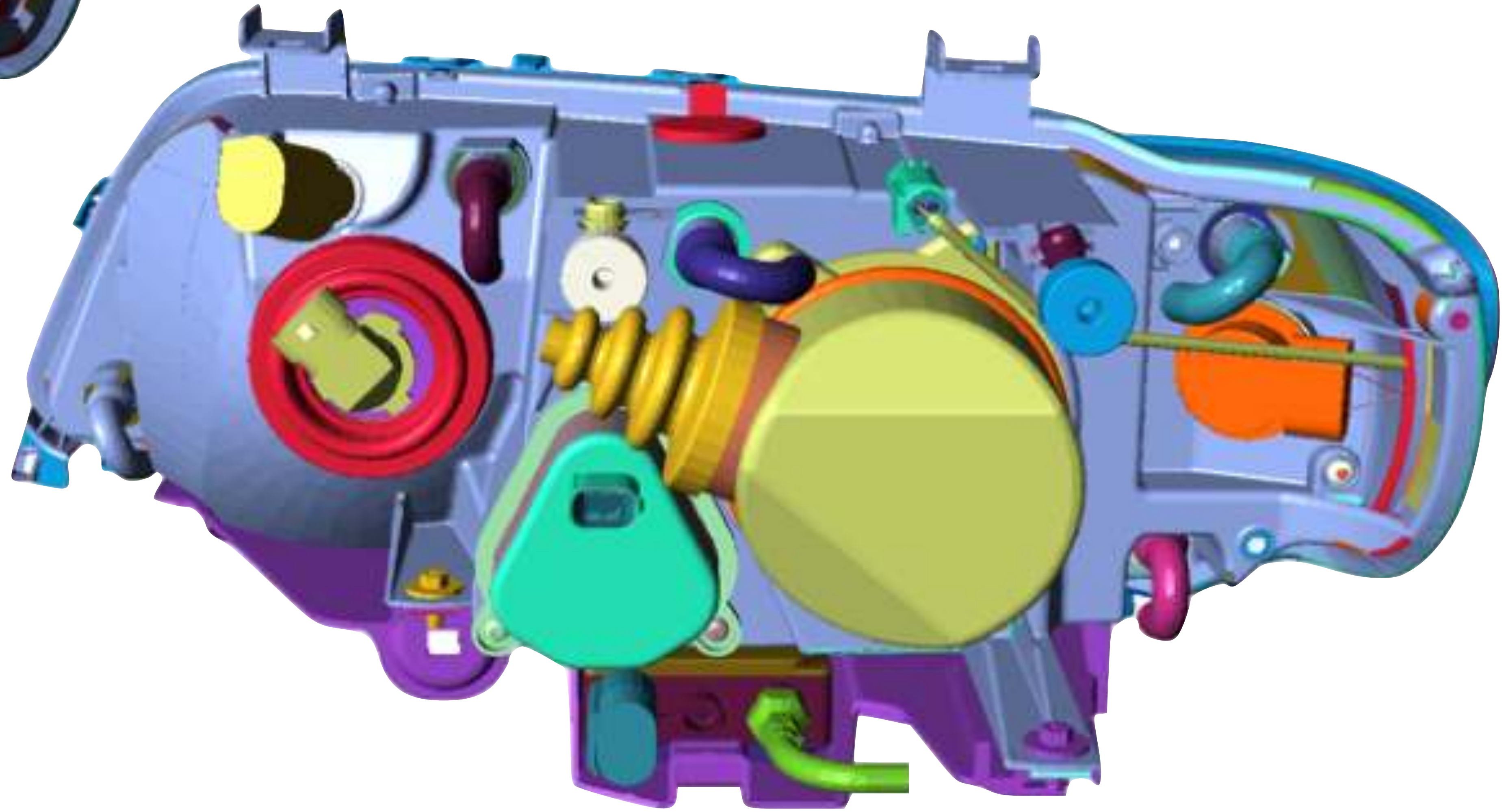


Results



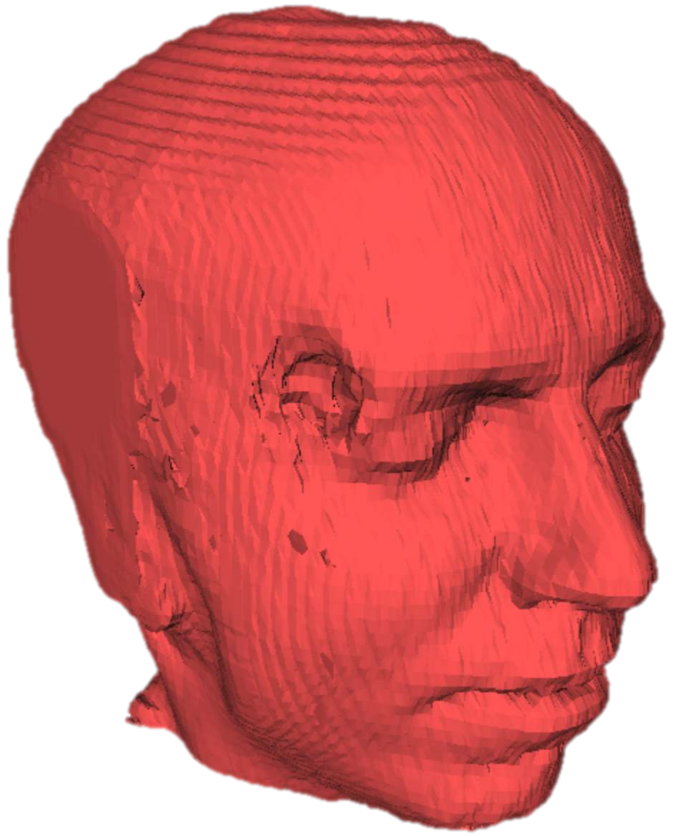
Before

After

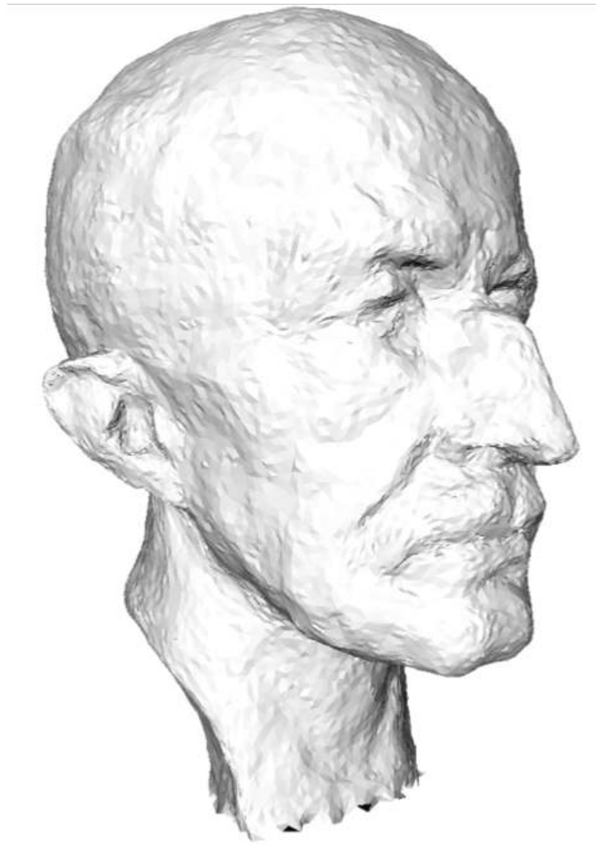


Mesh Smoothing

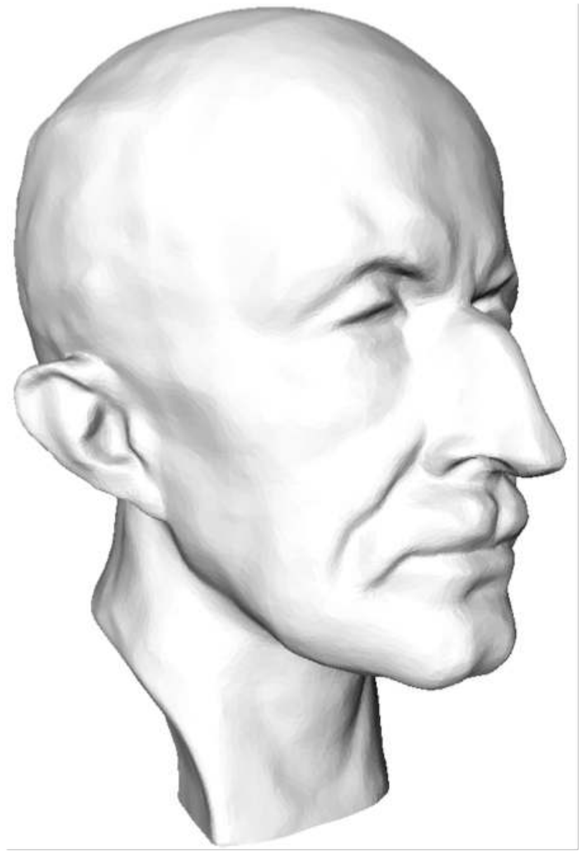
- Frequent problem: meshes are noisy (e.g., from marching cubes, or point cloud reconstruction)



Typical output of marching cubes



Output from laser scanner after meshing



Desired, smoothed mesh

- Idea: "convolve" mesh with a filter (kernel), like Gaussian filter for images

Digression/Recap: Image Smoothing (Blurring)

- Simple, linear filtering by convolution:
  - $I = I(x,y)$ = input image, $J = J(x,y)$ = output image

$$J(x,y) = \sum_{\substack{i=-k,\ldots,+k \\ j=-k,\ldots,+k}} I(x+i, y+j) H(i,j)$$

  - H is called a kernel, k = kernel width
- Sequential algorithm to construct J:
  - Slide a $k \times k$ window across I
  - At every pixel of I, compute weighted average of I inside window, weighted by H

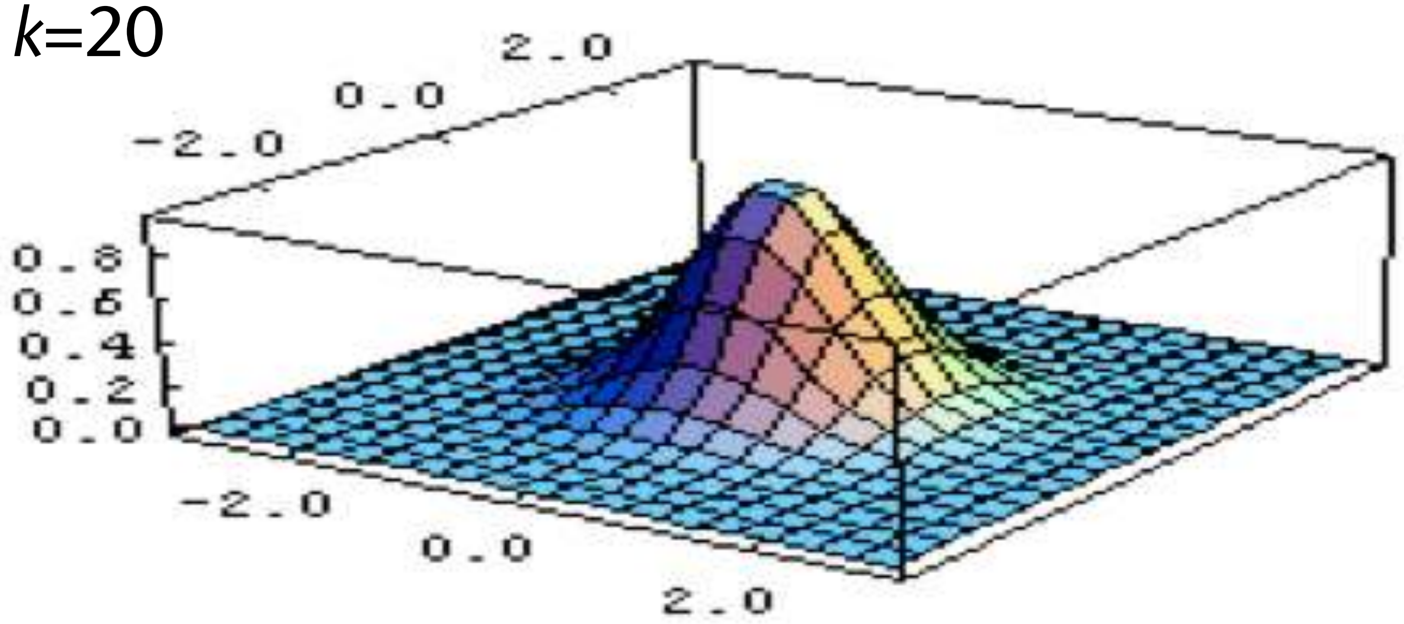
Examples

- Gaussian kernel

k=3

$$H = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

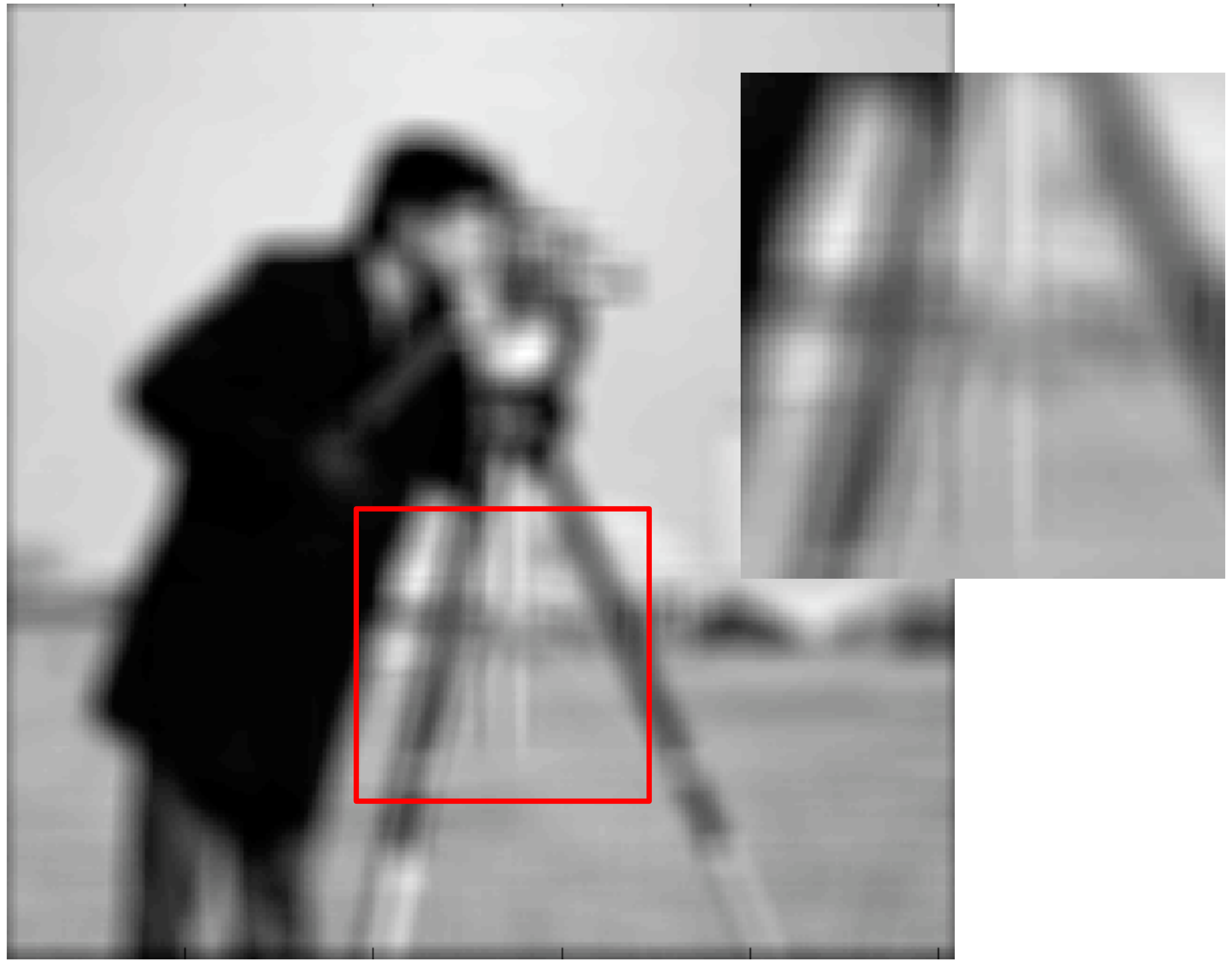
k=20



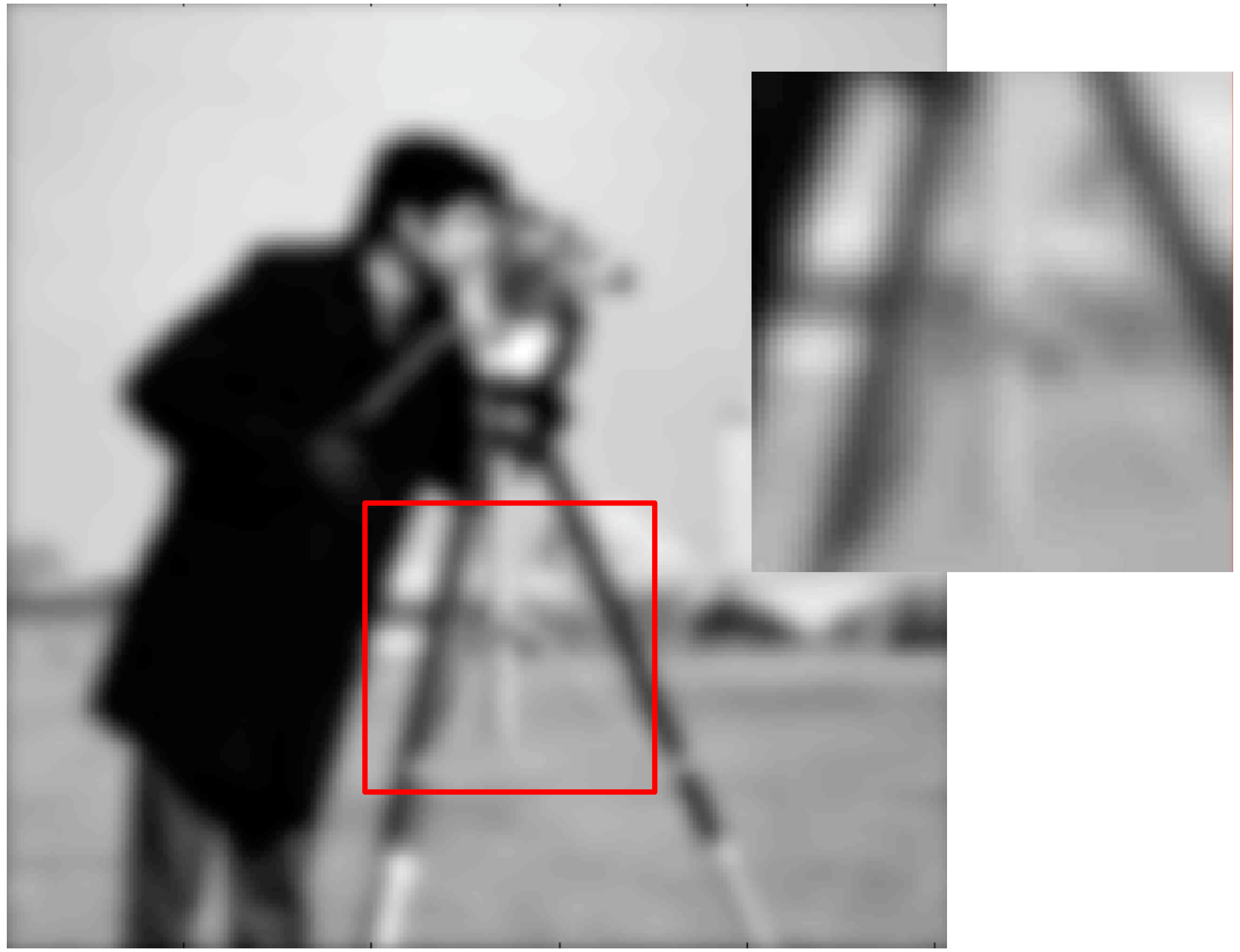
- Box filter (= simple averaging):

$$H = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

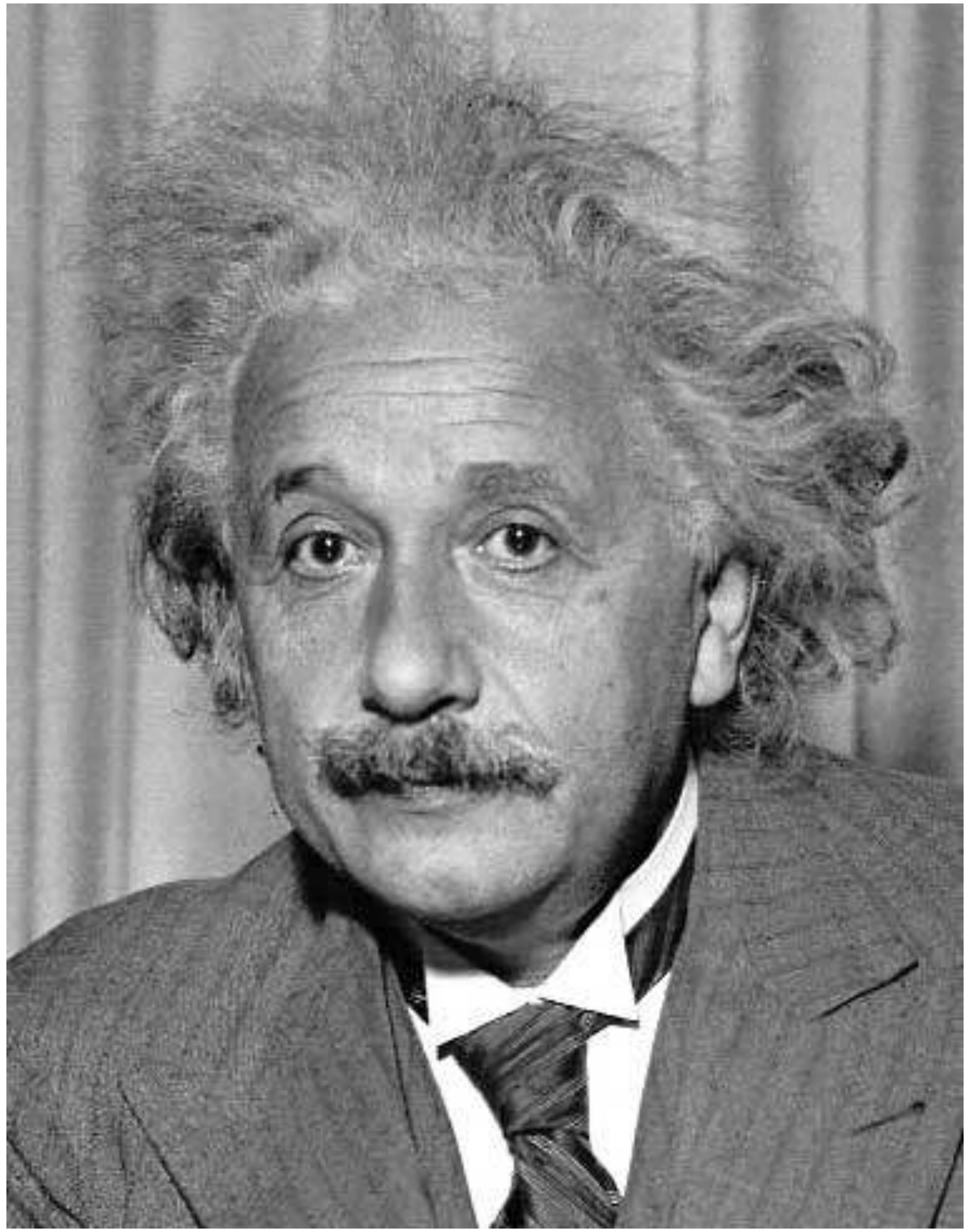
Box



Gaussian

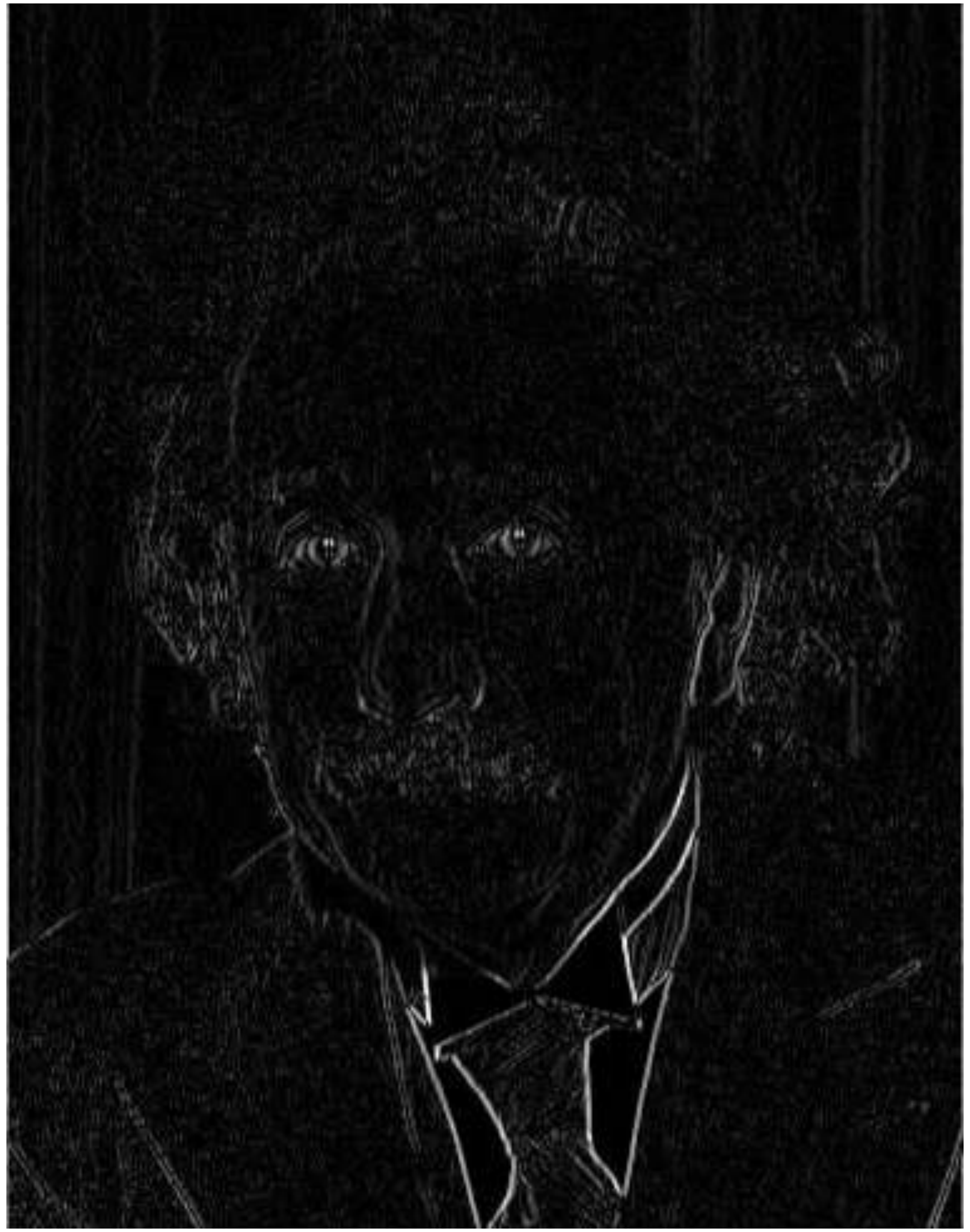


Digression: Edge Extraction

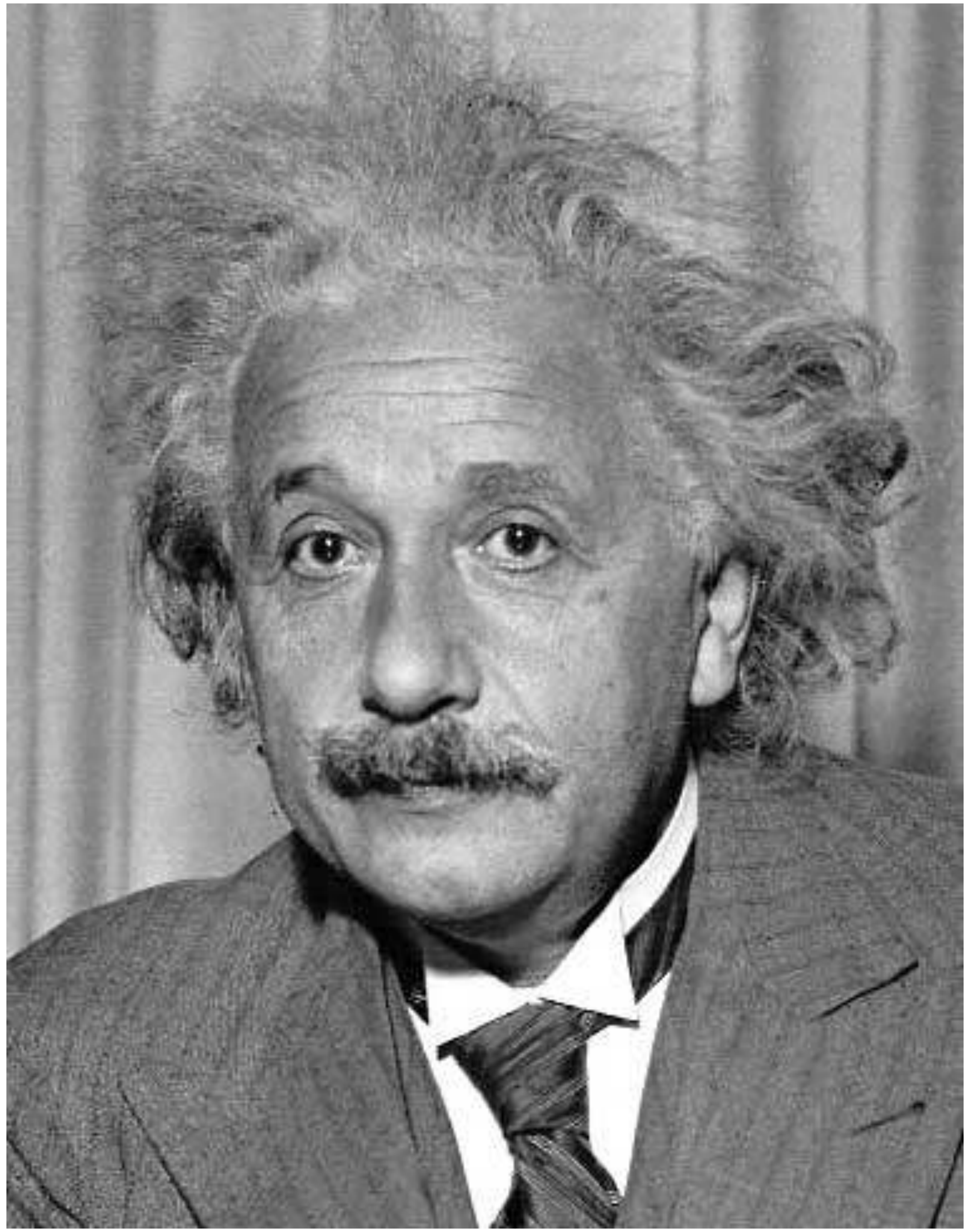


Vertical Sobel Operator

$$\begin{matrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{matrix}$$

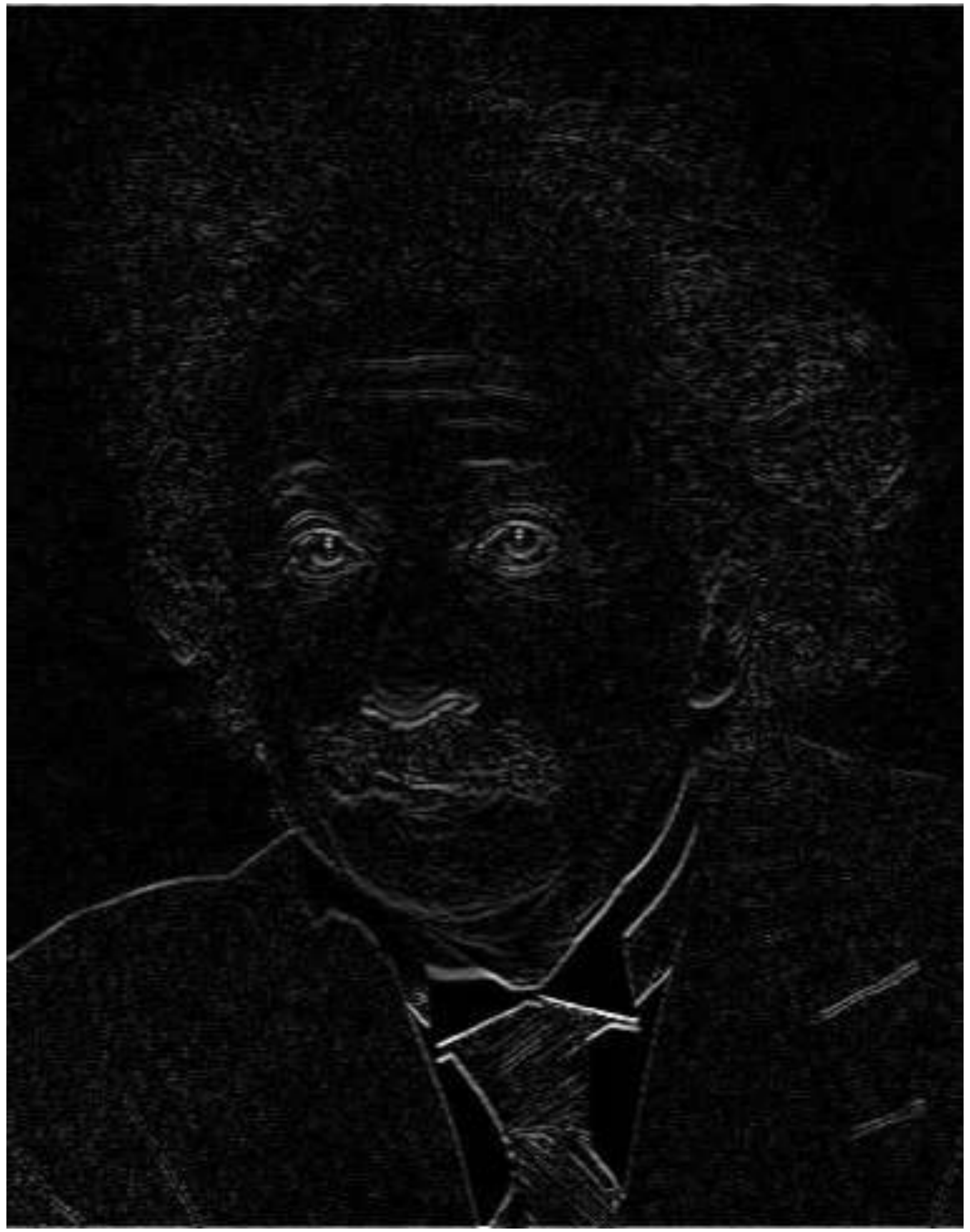


Vertical edges (absolute value)



Horizontal Sobel Operator

$$\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

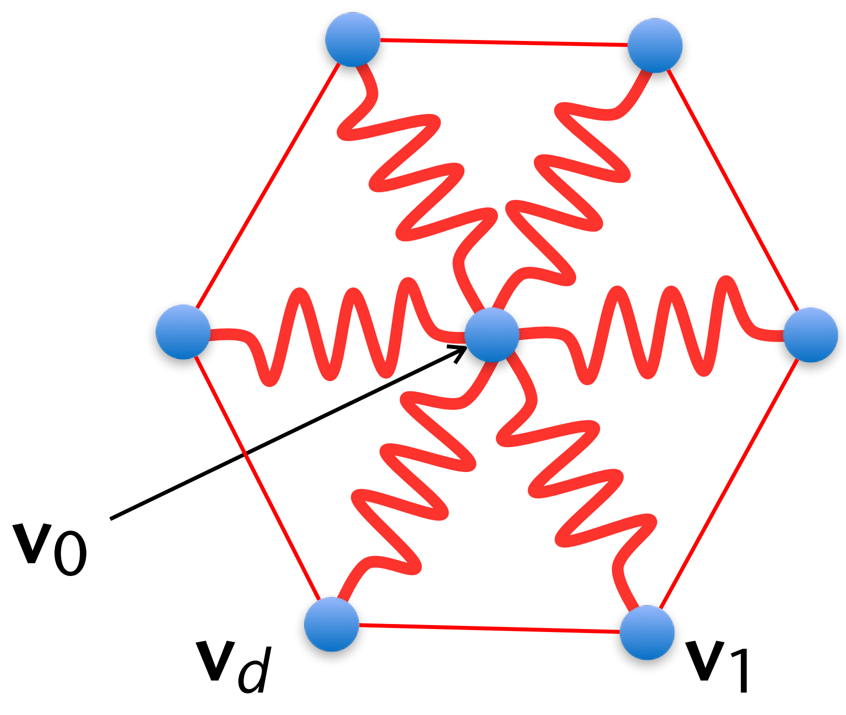


Horizontal edges (absolute value)

- Problem: we can't simply apply the convolution idea to meshes!
- Why not?
- Meshes don't have a canonical , tensor-structure-like parameterization!
    - I.e., usually there is no parameterization like x and y in the plane
- Goal: filter *without* parameterization

Laplacian Smoothing

- Idea:
  - Consider edges as springs
  - For a vertex $\mathbf{v}_0$, determine its position of *least* energy within its 1-ring

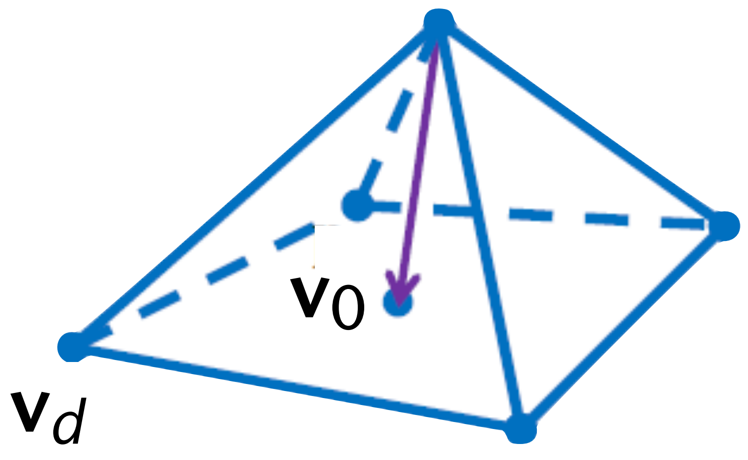


- Energy of $\mathbf{v}_0$: $$E = \frac{1}{2} \sum_{i=1}^{d} \left\| \mathbf{v}_i - \mathbf{v}_0 \right\|^2$$
- Necessary condition for minimum: derivative equals zero

$$\frac{\mathrm{d}E}{\mathrm{d}\mathbf{v}_0} = \sum_{i=1}^{d} (\mathbf{v}_i - \mathbf{v}_0) = 0$$

- Iterative procedure: $$\mathbf{v}_0' = \frac{1}{d} \sum_{i=1}^{d} \mathbf{v}_i$$

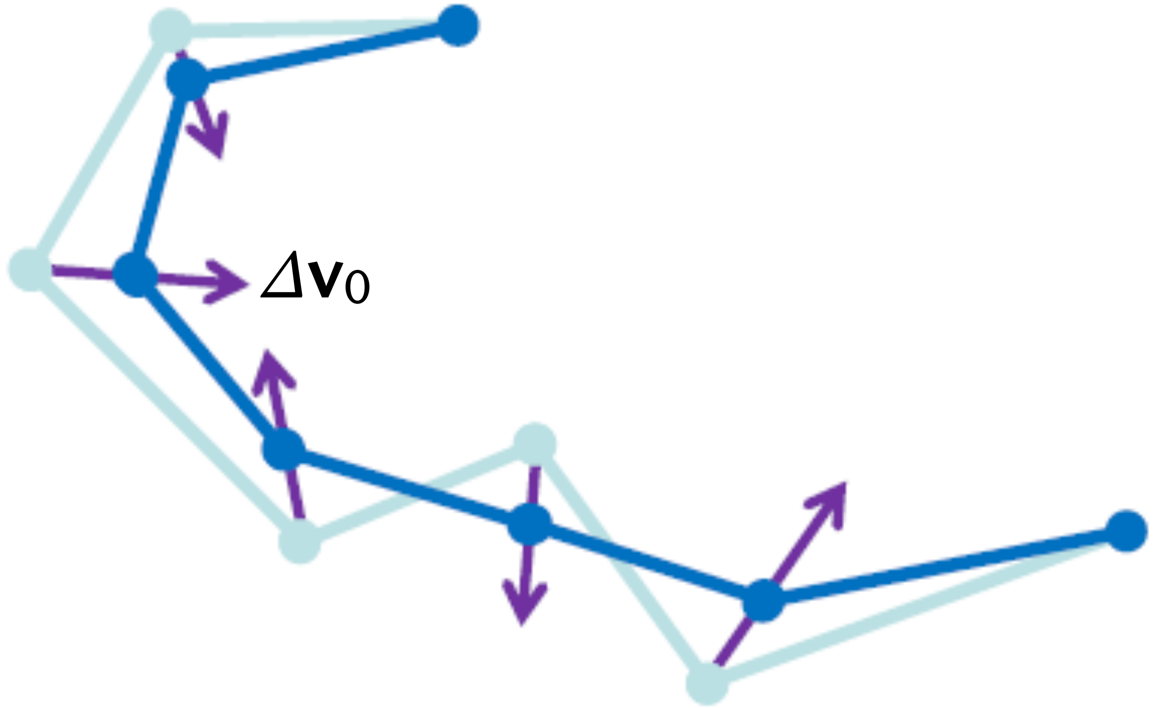
Sometimes a.k.a. "umbrella operator"



- Generalization: introduce "influence" of adjacent vertices and "speed"

$$\Delta \mathbf{v}_0 = \sum_{i=1}^{k} w_i(\mathbf{v}_i - \mathbf{v}_0) \ , \qquad \text{with } \sum w_i = 1 \ , \ w_i \geq 0$$

$$\mathbf{v}_0' = \mathbf{v}_0 + \lambda \Delta \mathbf{v}_0$$



- Simplest form of the weights:

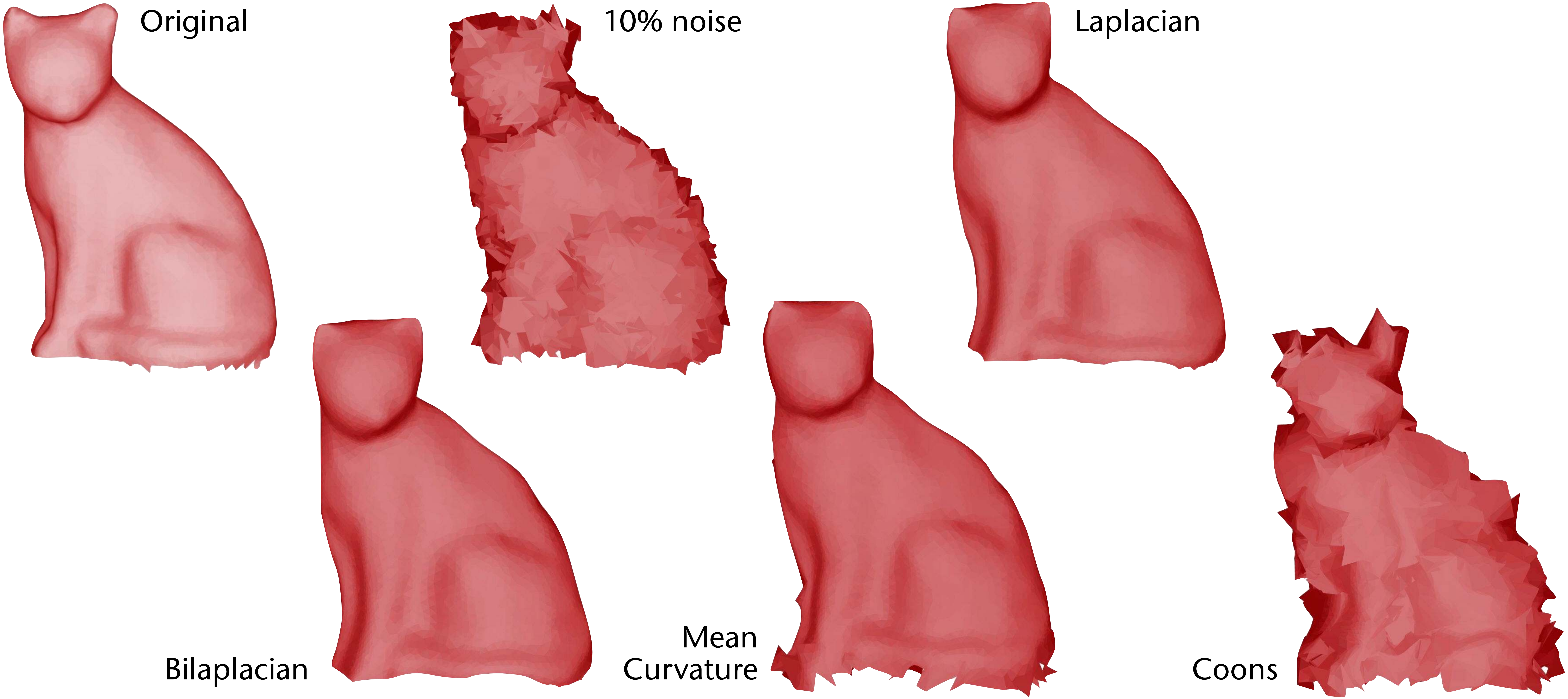
$$\Delta \mathbf{v}_0 = \frac{1}{d} \sum_{i=1}^{d} (\mathbf{v}_i - \mathbf{v}_0)$$

where d = degree of $\mathbf{v}_0$ = number of neighbors

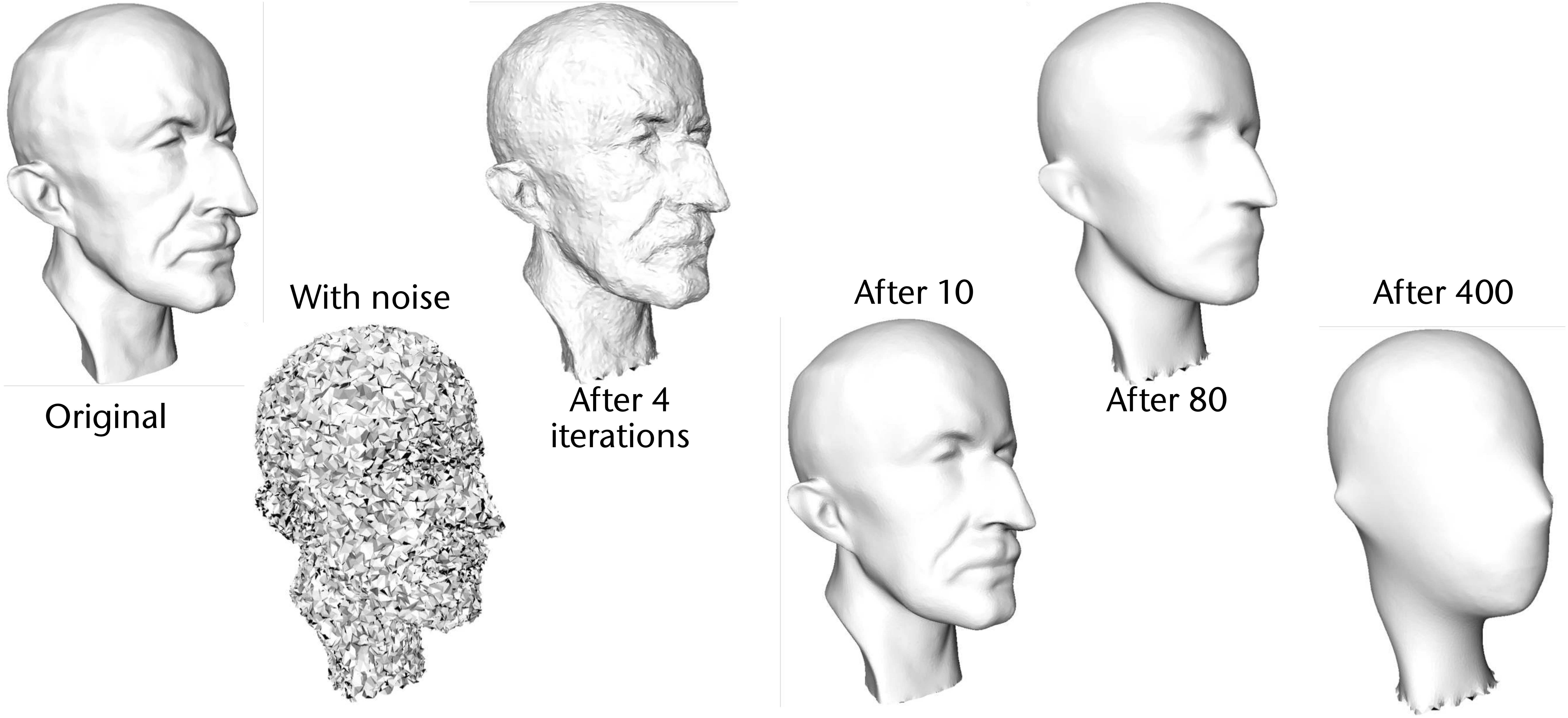
- Better weights are $w_i = \frac{1}{\|\mathbf{v}_i - \mathbf{v}_0\|}$ or $w_i = e^{-\|\mathbf{v}_i - \mathbf{v}_0\|^2}$ ("better" by experiment)

(see chapter "Object Representations" for more)

Comparison with Other Smoothing Operators (not presented here)



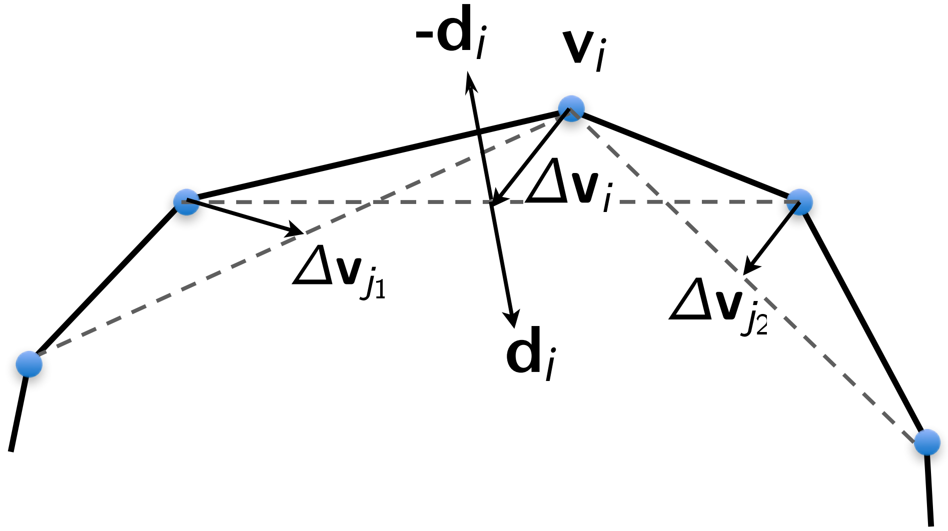
Problem: Laplace-Smoothing Causes Shrinking



A Simple Extension to Prevent Shrinking

- Like before, for every $\mathbf{v}_i$ compute

$$\Delta\mathbf{v}_i = \frac{1}{d} \sum_{j \in \mathcal{N}(i)} (\mathbf{v}_j - \mathbf{v}_i)$$



- Average all neighboring Δ's (*including* the own Δ):

$$\mathbf{d}_i = \frac{1}{d+1} \sum_{j \in \mathcal{N}(i) \cup i} \Delta\mathbf{v}_j$$

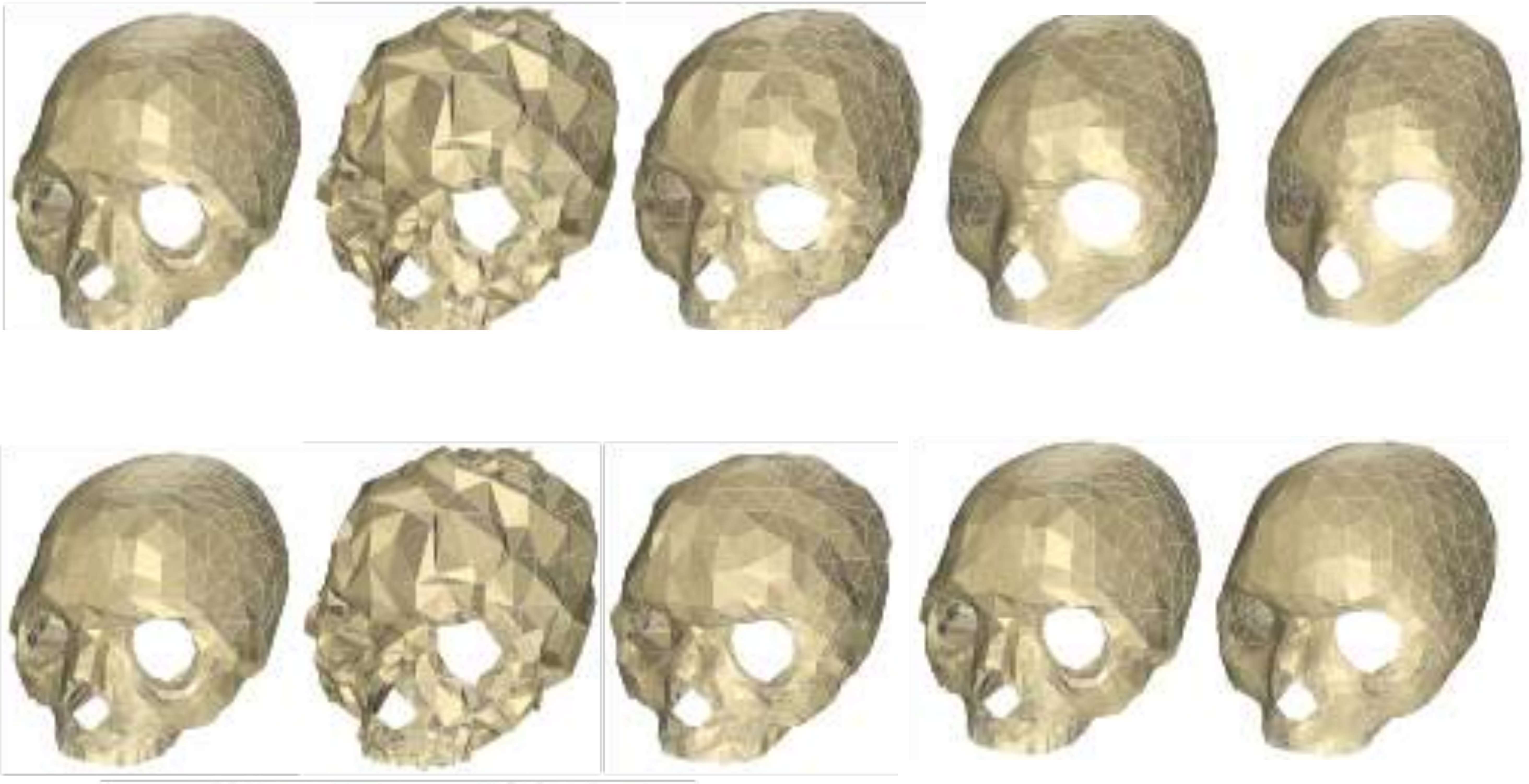
- Push the new vertex towards the 1-ring equilibrium *and outwards* away from the local direction of contraction ($\mathbf{d}_i$):

$$\mathbf{v}_i' = \mathbf{v}_i + \lambda\big(\alpha\Delta\mathbf{v}_i - (1-\alpha)\mathbf{d}_i\big)$$

Comparison

Laplacian smoothing

Smoothing with pushback



Global Laplacian Smoothing

- Given: mesh $M = (V, E, F)$, $V = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, $\mathbf{v}_i = (x_i, y_i, z_i)$
- Sought: mesh M' with vertices $\mathbf{v}_i'$ such that
  - M' is smoother than M, and
  - M' approximates M
- If M' was perfectly smooth (i.e., a plane), we could find weights s.t.

$$\forall i : \sum_{j \in \mathcal{N}(\mathbf{v}_i')} w_{ij}(\mathbf{v}_j' - \mathbf{v}_i') = 0 \qquad (1)$$

- This can be written as 3 systems of linear equations, one for x coords, one for y coords, one for z
  - In the following, we will deal with the x coords – y and z work similarly

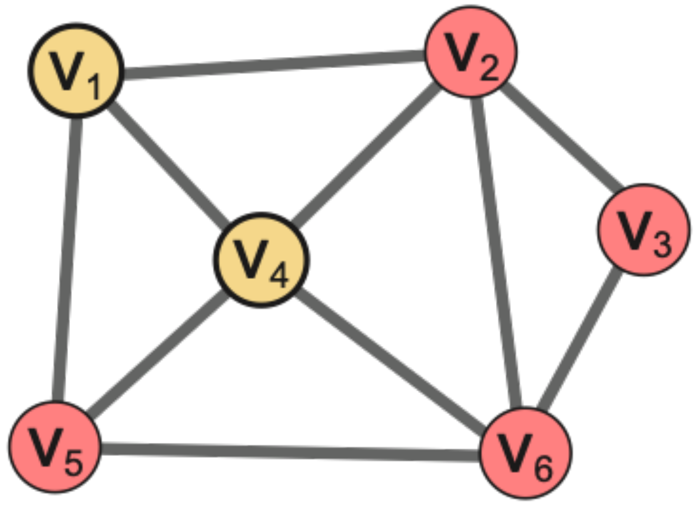
- Consider the x coords; write (1) as $\mathbf{L} \begin{pmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{pmatrix} = 0$

where **L** is a $n \times n$ matrix, with $L_{ij} = \begin{cases} -1 & , i = j \\ w_{ij} & , (i,j) \in E \\ 0 & , \text{else} \end{cases}$

- Definition: **L** is called the Laplacian of the mesh
  - In a sense, **L** encodes the adjacency of the mesh
- Analogously, construct a system of equations of y and z

- Example: for sake of simplicity, use $w_{ij} = \frac{1}{d_i}$

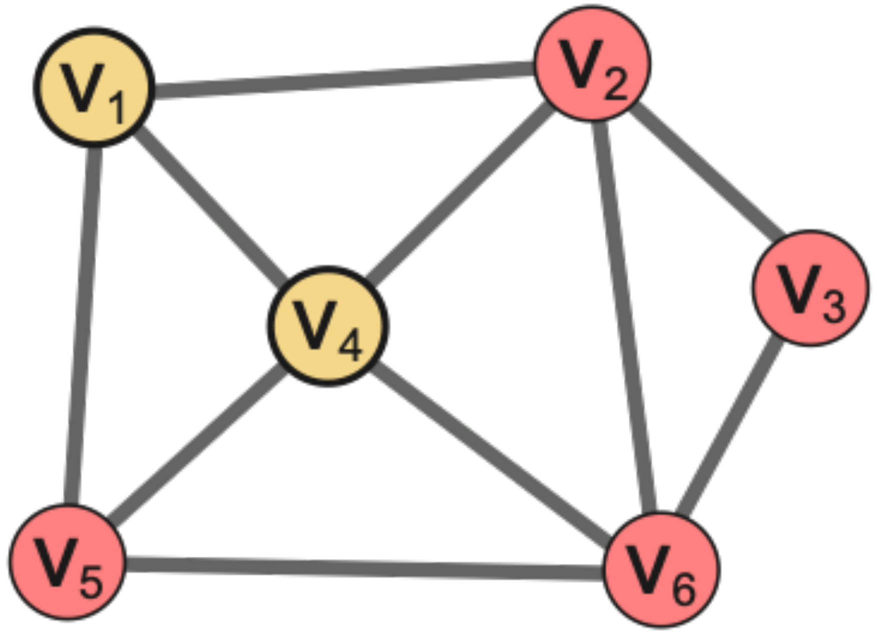
$$\mathbf{L} = \begin{pmatrix} -1 & 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/4 & -1 & 1/4 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & -1 & 0 & 0 & 1/2 \\ 1/4 & 1/4 & 0 & -1 & 1/4 & 1/4 \\ 1/3 & 0 & 0 & 1/3 & -1 & 1/3 \\ 0 & 1/4 & 1/4 & 1/4 & 1/4 & -1 \end{pmatrix}$$



- Warning: **L** has rank *n*-1, *n* = # vertices
- "Proof" by example: vector **x** = (1, ..., 1)^T is a solution to $\mathbf{L}\mathbf{x} = 0$ (and for all α , $\mathbf{L}(\alpha\mathbf{x}) = 0$, too)
  - Check for yourself: ist that so?

- Solution: "anchor" one vertex, i.e., fix its position
- For instance, in our example, add condition $\mathbf{v}'_1 = \mathbf{v}_1$:

$$\begin{pmatrix} -1 & 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/4 & -1 & 1/4 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & -1 & 0 & 0 & 1/2 \\ 1/4 & 1/4 & 0 & -1 & 1/4 & 1/4 \\ 1/3 & 0 & 0 & 1/3 & -1 & 1/3 \\ 0 & 1/4 & 1/4 & 1/4 & 1/4 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ x_1 \end{pmatrix}$$



- This system now has a unique solution

- Avoiding shrinking: introduce another constraint requiring the barycenters of the new triangles be the same as the barycenters of the old ones

$$\forall(i,j,k) \in F : \frac{1}{3}(\mathbf{v}'_i + \mathbf{v}'_j + \mathbf{v}'_k) = \frac{1}{3}(\mathbf{v}_i + \mathbf{v}_j + \mathbf{v}_k) \qquad (2)$$

- Write (1) and (2) as

$$\begin{pmatrix} \mathbf{L} \\ \mathbf{B} \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{b} \end{pmatrix} \qquad (3)$$

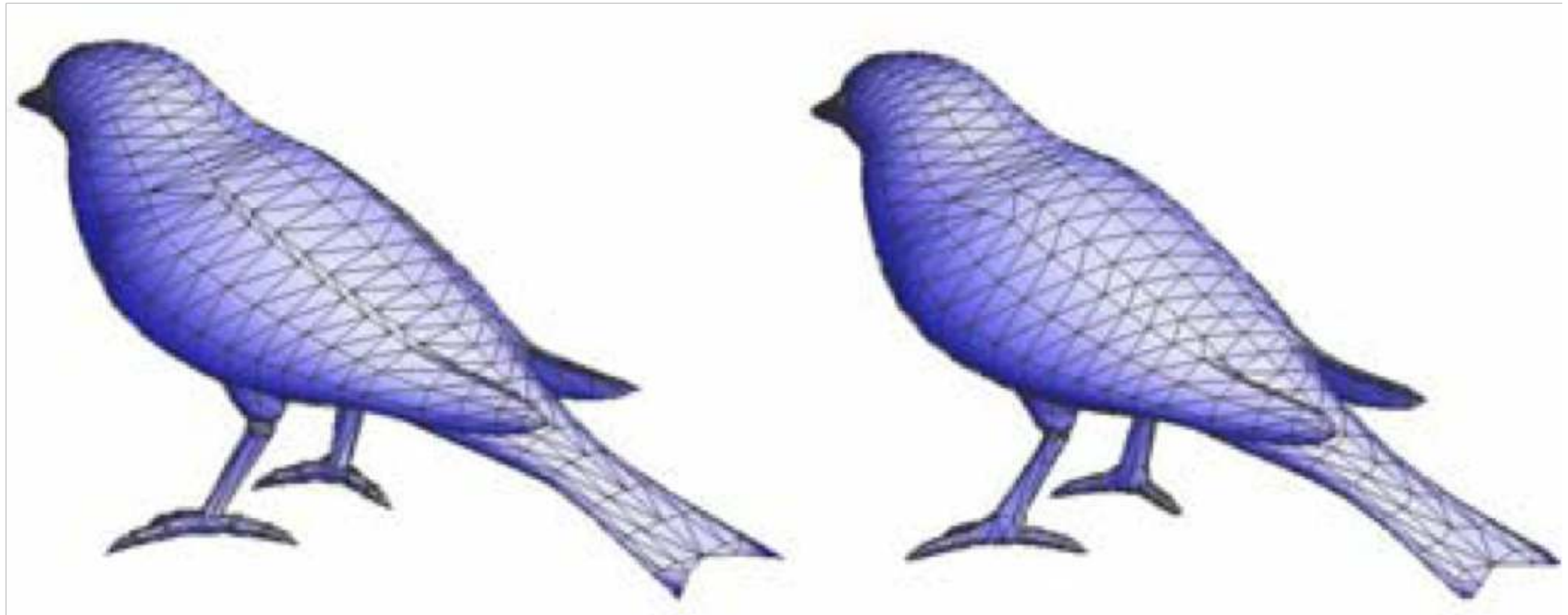
where **B** is a $m \times n$ matrix, m = number of triangles, and **b** is a column vector with m entries, where the k-th row corresponds to triangle $F_k = (i_1, i_2, i_3)$ and $B_{ki} = \frac{1}{3}$, for $i = i_1, i_2, i_3$, 0 elsewhere, and $b_k = \frac{1}{3}(x_{i1} + x_{i2} + x_{i3})$

- Solve (over-determined) system (3), which has the form $\mathbf{Ax} = \mathbf{c}$ in the least squares sense:

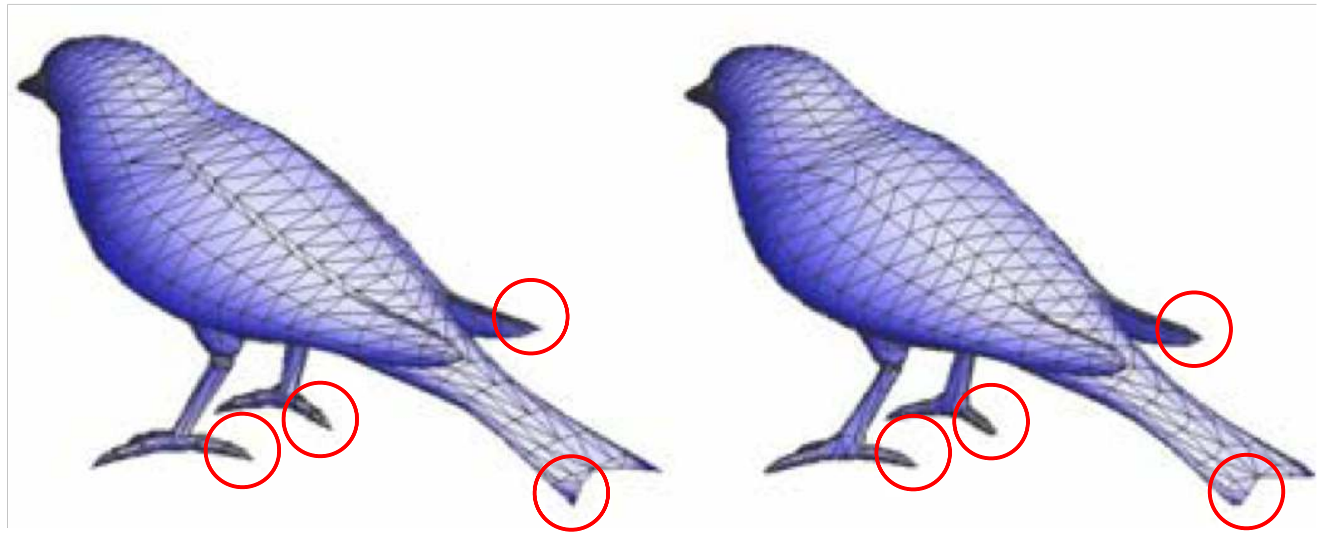
$$\mathbf{x} = (\mathbf{A}^{\mathsf{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathsf{T}}\mathbf{c}$$

  - In real life, use a sparse solver, e.g., TAUCS or OpenNL

- Results:



Original   Smoothed



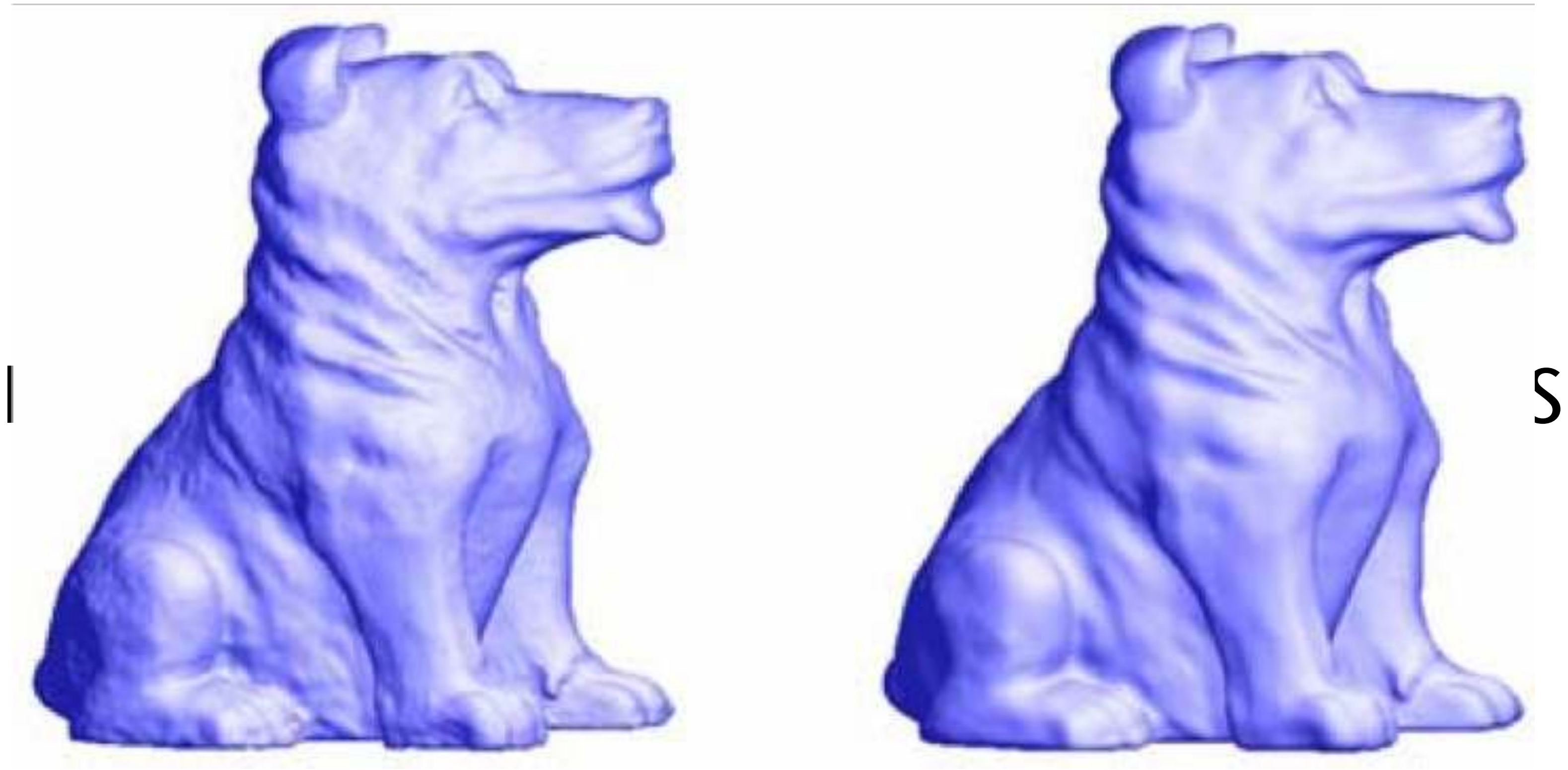
- Further requirement: certain points ("features") should be maintained
- Solution: introduce more constraints
  - Pick feature points $\mathbf{v}_{i_1}, \ldots, \mathbf{v}_{i_k}$
    - Either by user, or by automatic salient point detectors
  - Add constraint $\mathbf{v}'_{i_l} = \mathbf{v}_{i_l}$, $l = 1, \ldots, k$ (4)
  - Add equations (4) to system (3):

$$\begin{pmatrix} \mathbf{L} \\ \mathbf{B} \\ \mathbf{C} \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix}$$

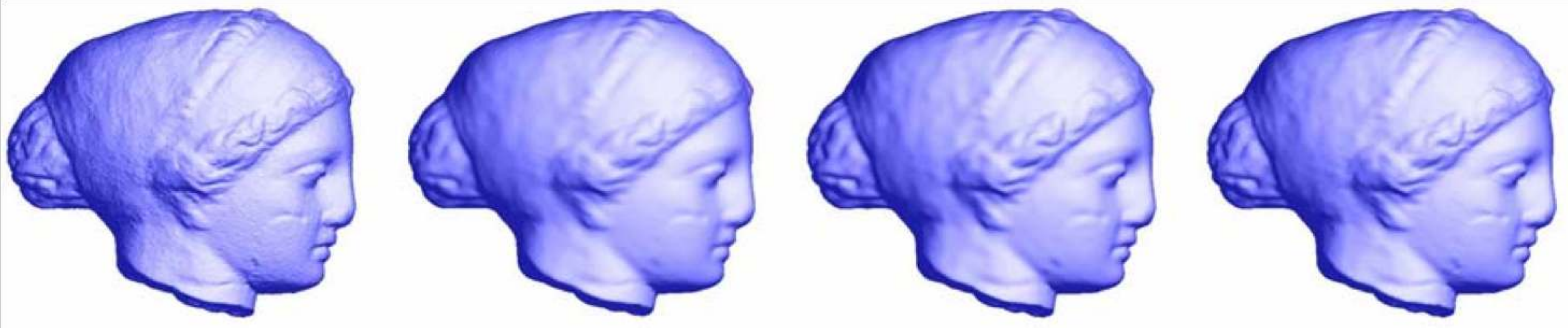
    where $\mathbf{C}$ is a matrix containing in every row l just one 1 at position i_l , $1 \le l \le k$, and $\mathbf{c} = (x_{i_1}, \ldots, x_{i_k})$
  - Again, we do this for x-, y-, and z-coordinates separately

Results

Noisy original



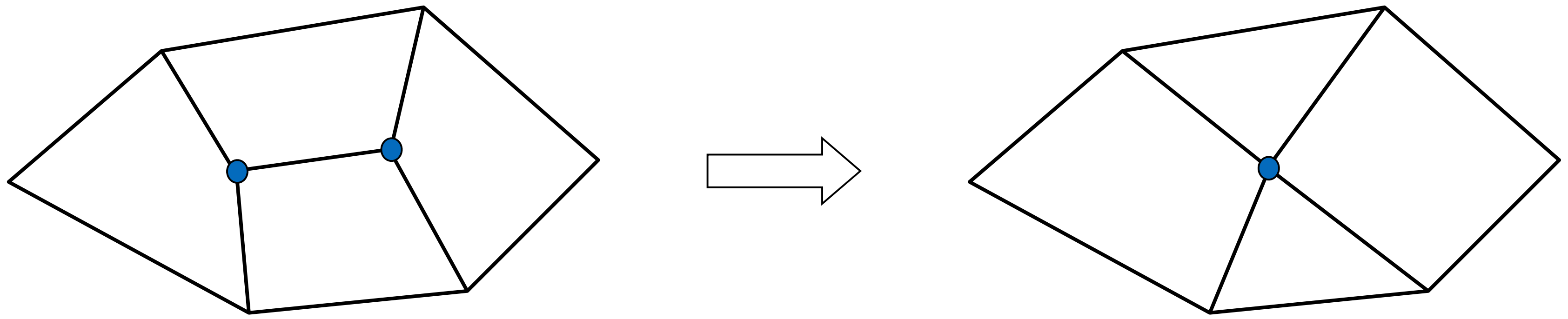
Smoothed



Noisy original | Laplacian smoothing | Bilateral smoothing | Global smoothing

Mesh Simplification

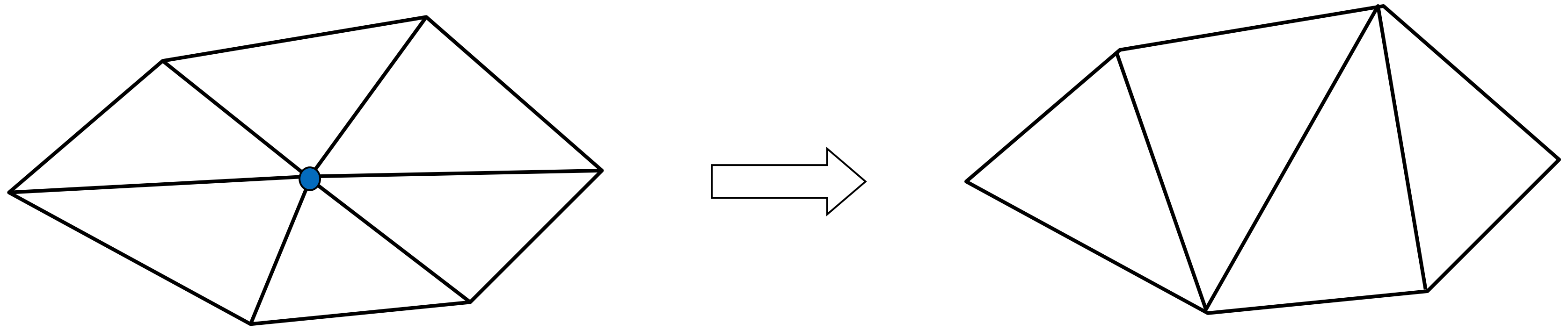
- Simplification: Generate a coarse mesh from a fine (hi-res) mesh
  - While maintaining certain criteria (will not be discussed further here)
- Elementary operations:
  - Edge collapse:



    - All edges adjacent to the edge are required

(More details in the course "Virtual Reality ..")

  - Vertex removal:



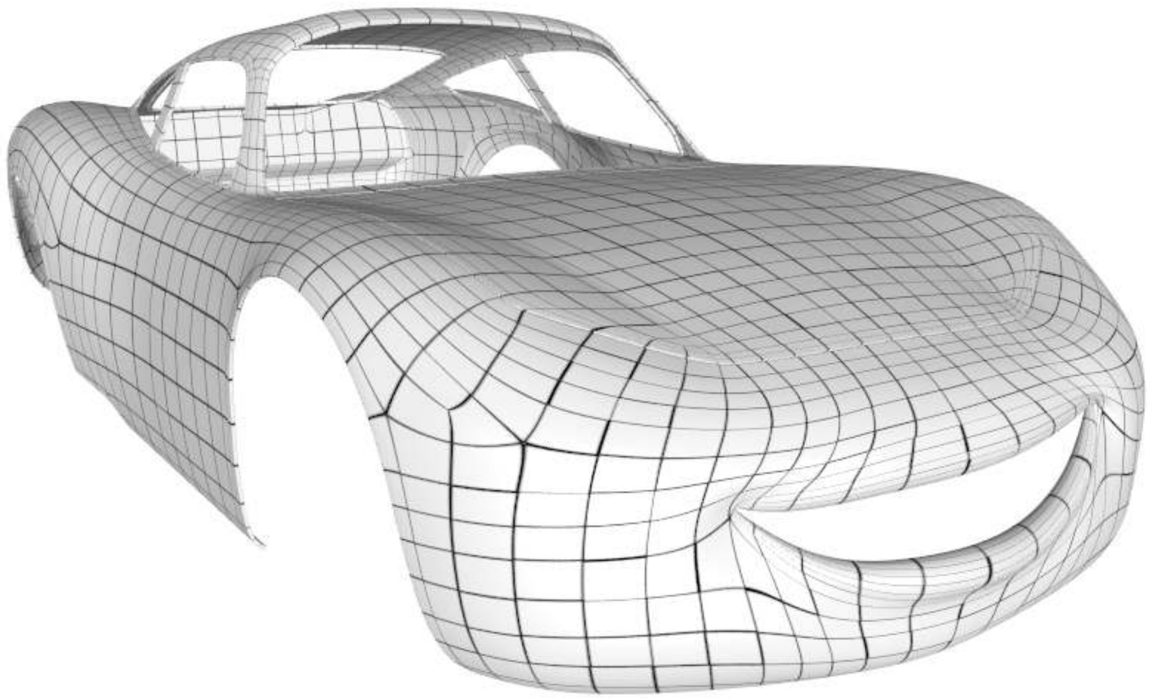
    - All edges incident to the vertex are needed

Subdivision Surfaces: One of the First Movies

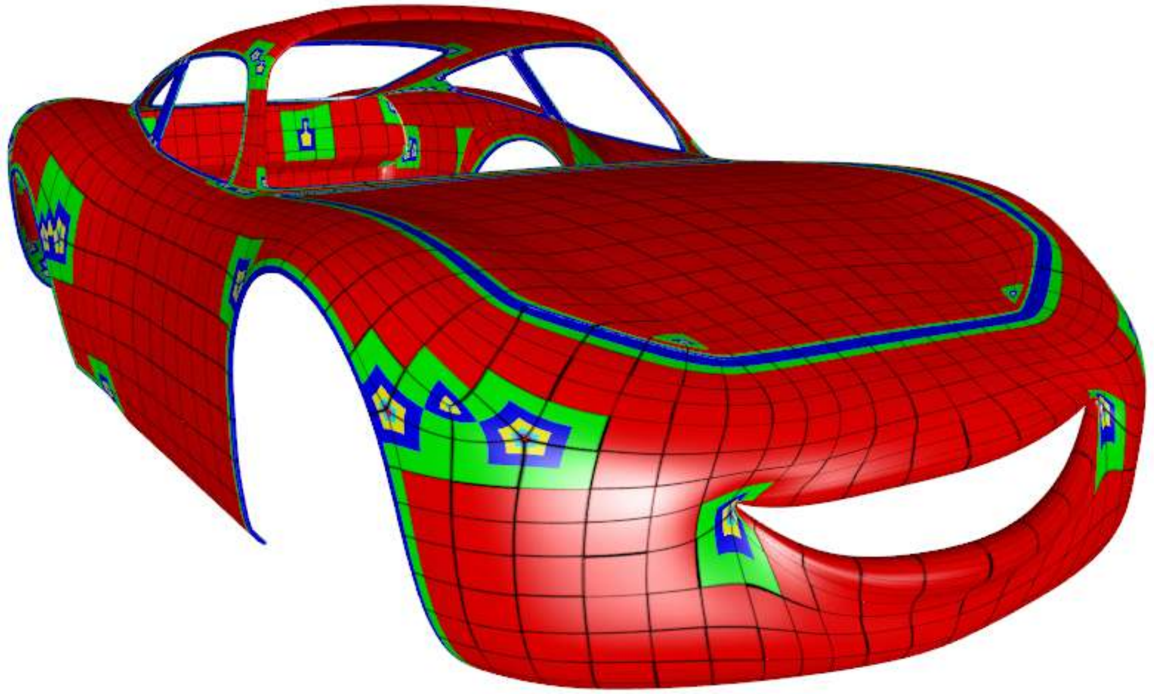
[Pixar: "Geri's Game"]

Examples from Animation Films

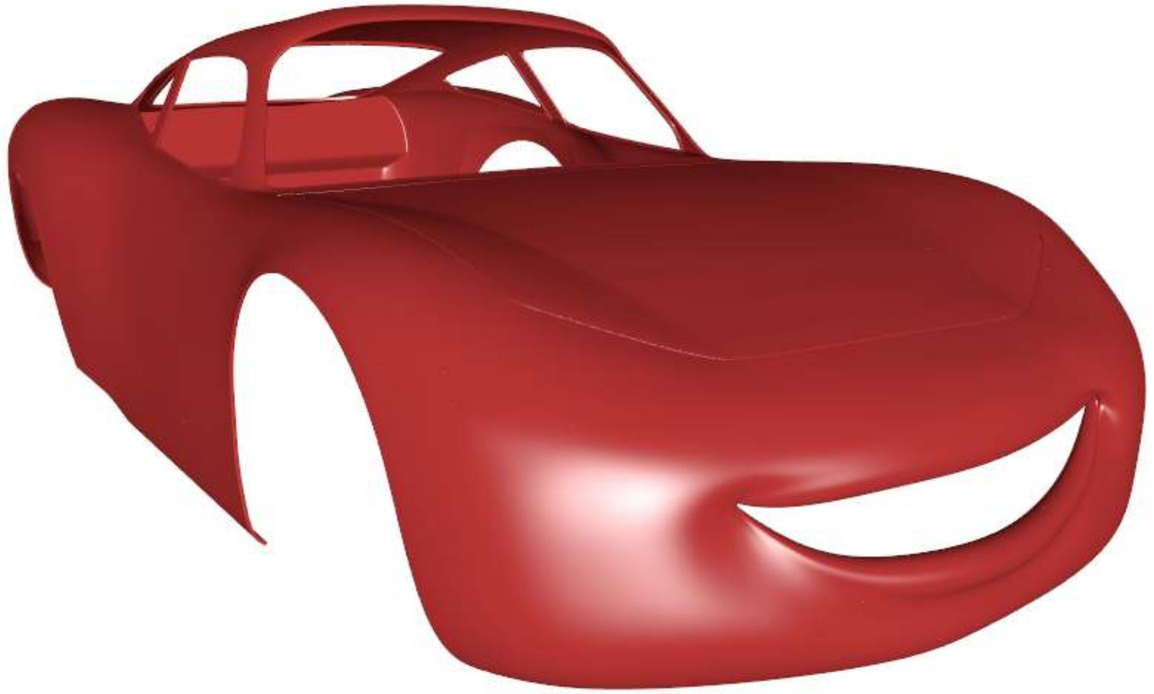
Input base mesh



Subdivision patch structure



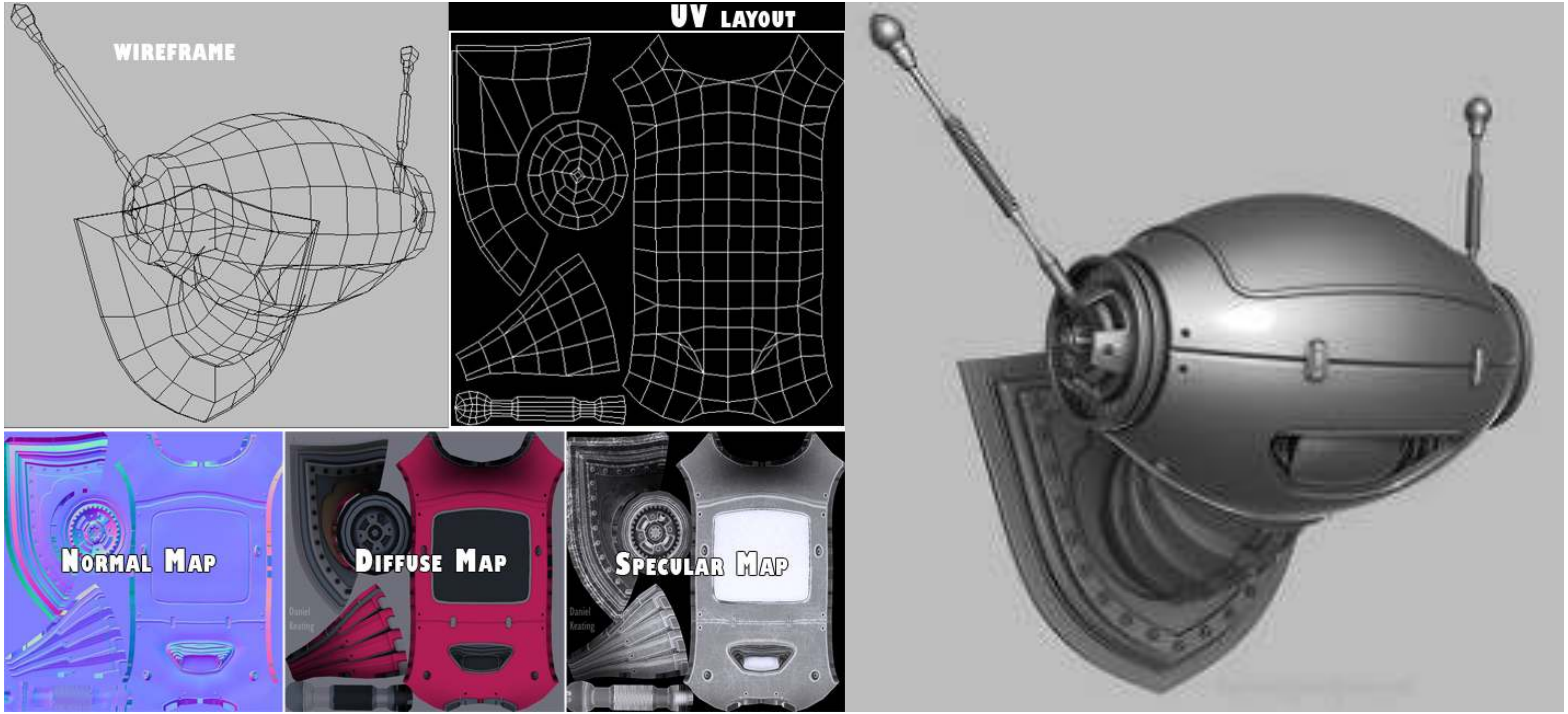
Final model



[Nießner et al., 2012]

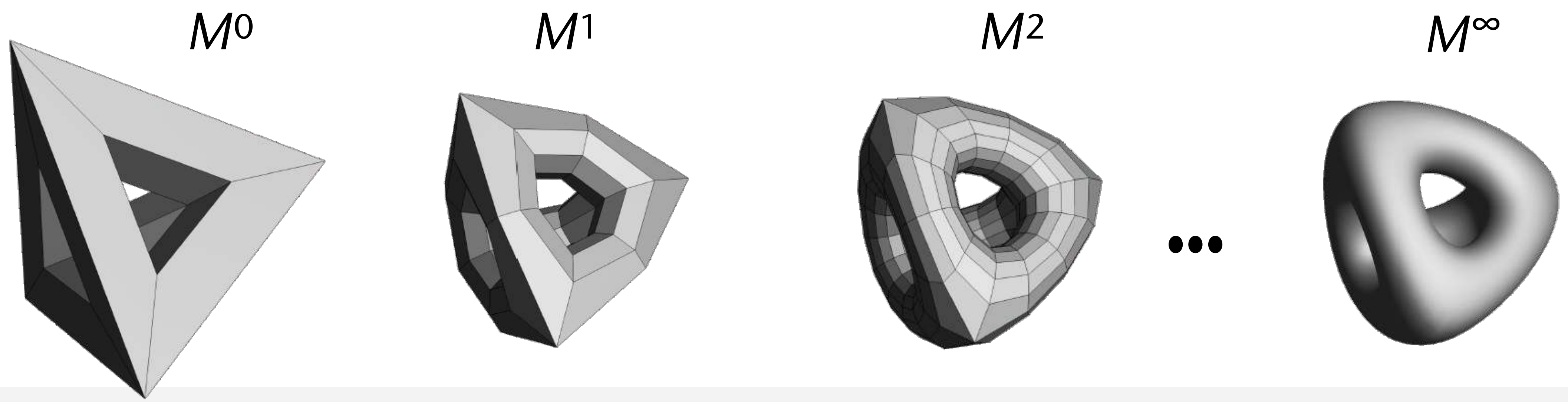
Example from Games

- Used to create high-poly models that are then used to bake texture maps (normal map, specular map, etc.) for the low-poly in-game models

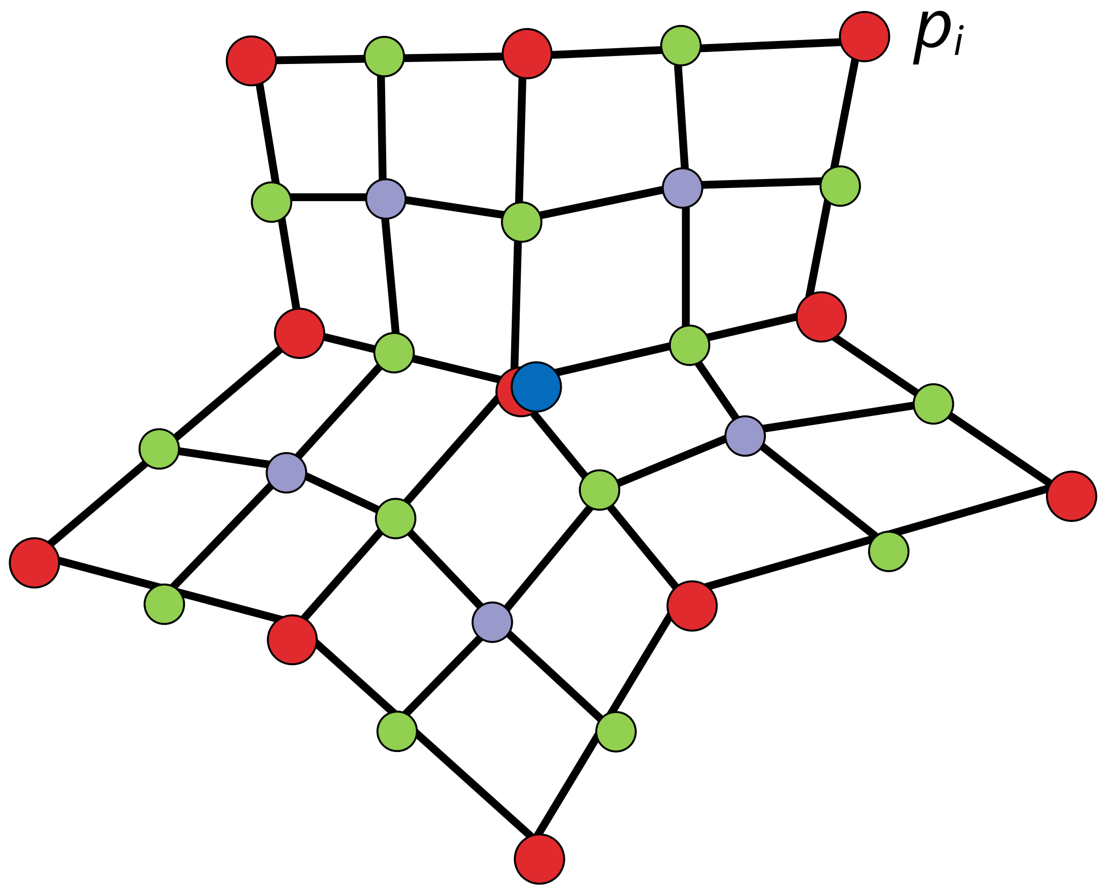


Basic Idea of Subdivision

- Start with a (simple) mesh M^0, called control mesh
- In each iteration i:
  1. Refinement: subdivide edges and faces of M^i
     - Some schemes split vertices ("dual" subdivision schemes)
  2. Weighted averaging: calculate new positions by averaging neighboring vertices
  - Results in a new mesh M^{i+1} (generation i+1)
- Ideally, the mesh converges to a limit surface



The Catmull-Clark Subdivision Scheme



- Let p_i = vertices of the "old" mesh generation
- For each face, calculate a new "face point"

$$f = \frac{1}{k}\sum_{i=1}^{k} p_i$$

- For each edge, calculate a new "edge point":

$$e = \frac{1}{4}(p_1 + p_2 + f_1 + f_2)$$

- For each old vertex, p, calculate a new"vertex point”:

$$p' = \frac{1}{m}q + \frac{2}{m}r + \frac{m-3}{m}p$$

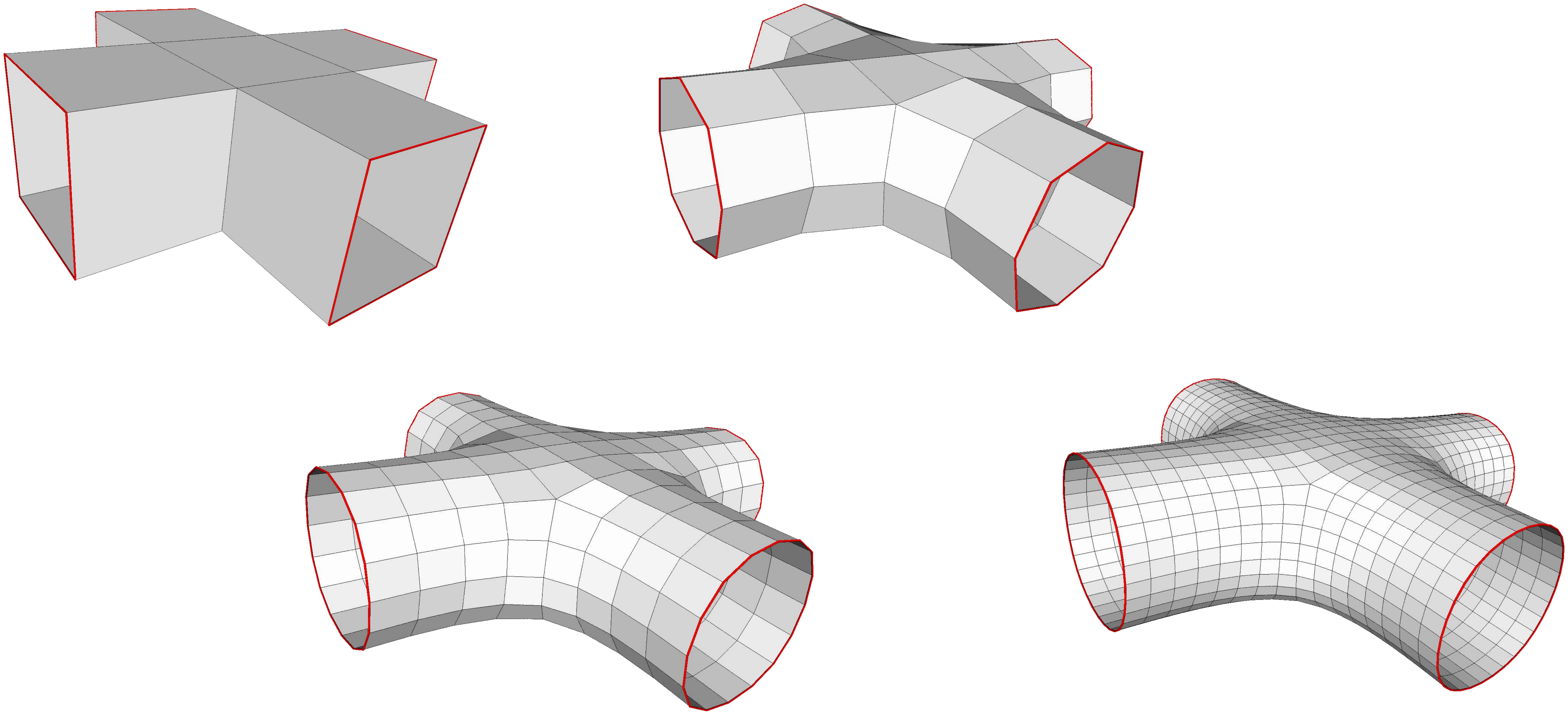
k = # old vertices incident to the face (valence)

p_1, p_2 = old vertices incident to the edge
f_1, f_2 = new face point of the faces incident to the edge

m = # faces/edges incident to old vertex (valence)
q = average of incident face points
r = average of incident edge points

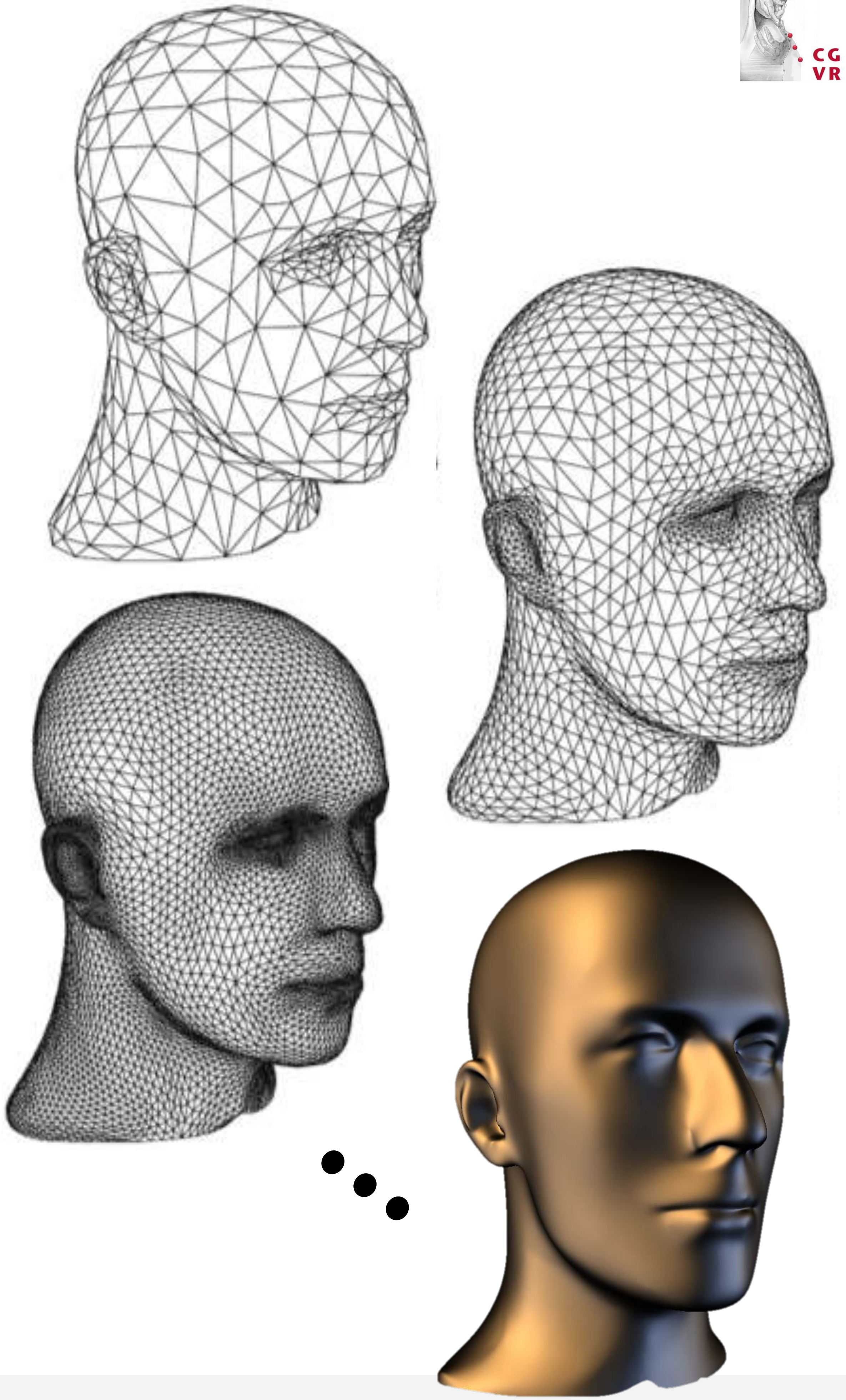
$$q = \frac{1}{m}\sum_{i=1}^{m} f_i \qquad r = \frac{1}{m}\sum_{i=1}^{m} e_i$$

Catmull-Clark in Action



Advantages

- Modelers and animators (artists) like object descriptions that are …
  - Easy to understand and control
  - Smooth, but creases can be added easily when needed
  - Offer different levels of detail, and LoD's can be made adaptive, e.g., view-dependent
  - Well-suited for animation, i.e., easy to deform
  - Allow for arbitrary topology (with holes and borders)
  - Compact (in terms of memory usage)



Subdivision Schemes (”Subdivision Zoo”)

Common schemes:

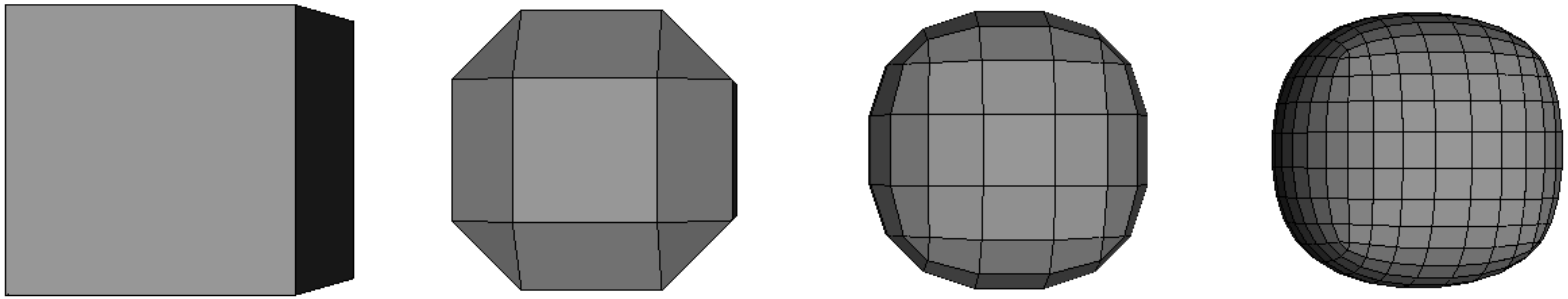
- Catmul Clark
- Doo-Sabin
- Loop
- Butterfly – Nira Dyn
- ...many more

Classification by:

- Mesh type: tris, quads, hex..., combination
- Face / vertex split (a.k.a. "primal" / "dual" scheme)
- Interpolating / Approximating
- Smoothness
- Linear/non-linear
- ...

Catmull-Clark vs Doo-Sabin

Doo-Sabin



Catmull-Clark